

Optimal exercise time to control glucose in type 2 diabetes

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Abstract

Diabetes is a lifelong disease that affect the way the body regulates blood glucose. Type 2 diabetes occurs when β cells do not produce enough insulin to regulate glucose in the blood. In this paper the authors used exercise as a tool to predict the optimal effect of glucose regulation in a diabetic patient using simulation. Computer simulations were performed on the proposed mathematical model to determine the number of heartbeats a diabetic patient need to train in other to regulate the glucose level to normalcy. This cross-sectional study took place at Sunyani Regional Hospital, one of the 10 regions in Ghana, between September 2018 and May 2019. The simulations show the effects of different rate of exercise on glucose level. The results shows that the optimal heartbeat for a diabetic patients is 140 beats per minute corresponding to a rate of 0.8. If this is done continuously for at least 3 times in a week, the glucose level could be regulated to normal within 50 weeks.

Keywords: Simulation; Exercise; Type 2 diabetes; Mathematical modeling

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

1 Introduction

Diabetes is a metabolic disorder that affects the body that causes blood glucose level to rise higher than normal. Type 2 diabetes is characterized by insulin resistance and more often it develops in adults, but children can also be affected. About 80% of the 371 million diabetics live in low and middle income countries, nearly five million deaths were due to diabetes and more than \$471 billion was spent on health care for diabetes in 2012 (IDF, 2012). The prevalence of diabetes has increased drastically in Africa. Ghana is one of the 32 countries of the IDF in African. 425 million people have diabetes in the world and more than 16 million people in African, it is estimated to be around 41 million by 2045. 518,400 cases of diabetes were recorded in Ghana in 2017 (IDF, 2017). There are challenges facing the fight of the disease such as lack of funding for noncommunicable diseases, lack of research available on diabetes, lack of availability of medications, differences in urban and rural

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patients, and inequity between public and private sector health care (Sonak *et al*, 2017). The research was done at the Sunyani Regional Hospital, the capital of Brong-Ahafo Region in Ghana. The regions demography includes these details: The region has a total of 690 health facilities comprising 29

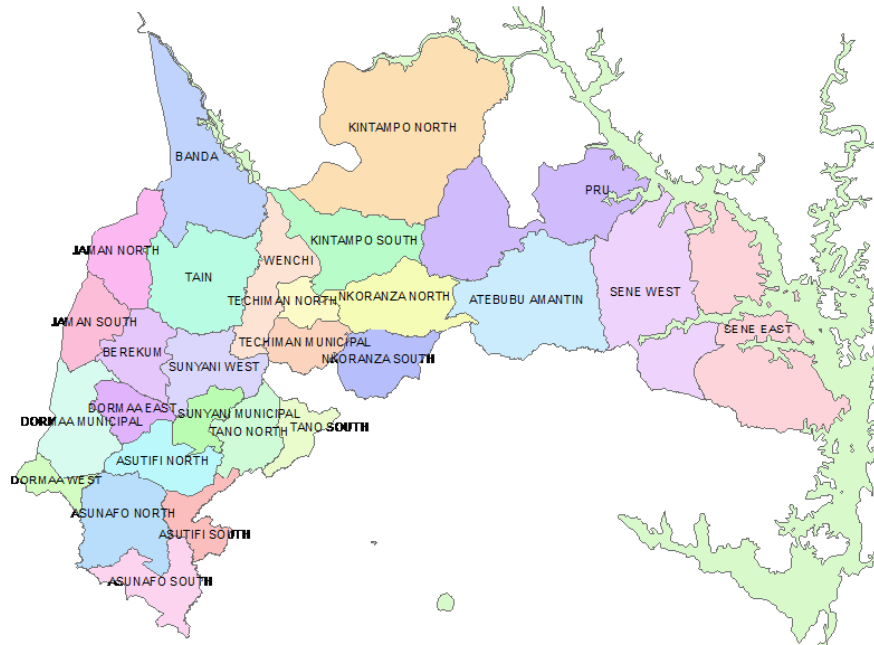


Figure 1: Map of Brong Ahafo Region of Ghana

Hospitals, 82 Health Centres, 112 Clinics, 43 private Maternity Homes and 423 Functional Community Health Planning Services (CHIPS). Research at the Sunyani Regional Hospital revealed that a high number of people in the Sunyani municipality have high blood pressure and high blood sugar levels but are not aware of their status. A total of 99 cases of diabetes were recorded at the hospital (Ghanaweb, 2003). Obesity is associated with diabetes among women in the region. 22 out of the 35 cases studied on epidemiological and clinical factors in diabetes patients at Kintampo in the region were females and 13 males. 41 to 50 age group had the peak incidence for females and from 51 to 60 age group for males (Ghanaweb, 2005).

2 Literature review

Mathematical models have been developed based on variables which include the concentrations of glucose in the blood, the concentration of glycogen in the liver, the concentration of the hormone glucagon, and the concentration of insulin in the blood. The main aim of diabetes treatment is to maintain the glucose level in the blood to a normal level. The ability to lower blood glucose depends on how the β cells responds to glucose (Bergman *et al*, 1981). (Chaikovska *et al*, 2015) developed mathematical models which were based on digestion of received glucose depending on the volume of consumed carbohydrate. The Levenberg-Marquardt method was used to identify the blood glucose dynamics. An increase in exercise intensity increases glucose uptake by the working tissues (Wasserman *et al*, 1991). (Ontan-Garca *et al*, 2013) worked in generating in-phase bursting electrical activity (BEA) in β cells with different behaviors such as active, inactive and continuous

spiking cells based on mathematical models using a discrete time coupling. Based on numerical simulations, synchronization in the insulin release is obtained from β cells with different behaviors. (Erica, 2012) also develop mathematical models based on insulin resistance and cell dysfunction. The determinants in cell failure was explored, from the perspective of endoplasmic reticulum stress and the unfolded protein response. It was concluded that with a basis for age related insulin resistance, nutrient plays an important role in health which helps in cell and molecule mechanisms. (Minghu, 2015) proposed three dynamic models to study the mechanism of glucose insulin regulatory system and the possible causes of diabetes. The progression of diabetes comes along with the apoptosis of pancreatic β cells. A dynamical system model is formulated based on physiology and studied by geometric singular perturbation theory. Liapunov function, Hopf bifurcation and backward bifurcation was used in the analysis. It was found that the intermittent rests of β cells in insulin secretion are essential for the cells to survive through the observation of the existence of a limit cycle.

3 Methodology

We will formulate a new type 2 diabetic model that involves exercise. The formulated model is in the form of a system of differential equations. This mathematical models has a compartmental structure $G(t)$, $X(t)$, $Y(t)$ and $Z(t)$ which depicts the flow of insulin and glucose between tissues such as the liver, muscle and heart. The model inputs are the Intravenous (IV) insulin infusion rate H and the rate of glucose absorption from the gut following a meal C . The model parameters v_n for $n = 1, \dots, 3$ is the transfer rate. The mathematical model developed from the schematic diagram in figure 2 is given below:

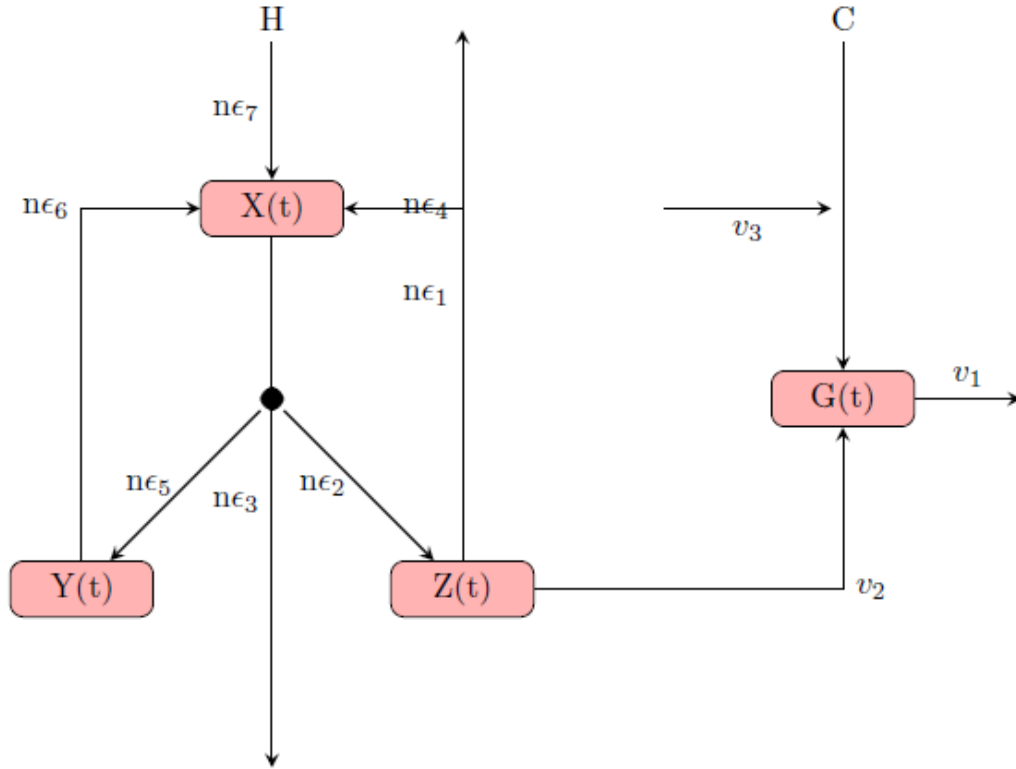


Figure 2: Insulin kinetics and glucose metabolism model with exercise

$$\frac{dG}{dt} = -(v_1 + nv_2Z(t))G(t)a_1 + \beta(t) + v_3 + C + rj_2j_3 - \gamma_1j_1j_4 - c_1j_1 - l_1 \quad (3.1)$$

$$\frac{dX}{dt} = nH\epsilon_7 + nZ(t)\epsilon_4 - nX(t)\epsilon_3 + \beta(t) - \gamma_2j_1j_4 - c_2j_4 - l_2 \quad (3.2)$$

$$\frac{dY}{dt} = nX(t)\epsilon_6 - nY(t)\epsilon_5 + \beta(t) - \gamma_2j_1j_4 - c_2j_4 - l_3 \quad (3.3)$$

$$\frac{dZ(t)}{dt} = nX(t)\epsilon_2 - nZ(t)\epsilon_1 + \beta(t) - \gamma_2j_1j_4 - c_2j_4 - l_4 \quad (3.4)$$

Table 1: Variable and descriptions

Symbol	Description	Parameter value	Source
$G(t)$	Plasma glucose concentration		
$X(t)$	Insulin mass in the blood		
$Y(t)$	Insulin mass in a remote compartment in fast equilibrium		
$\beta(t)$	Represents the β cells		
$Z(t)$	Insulin mass in a remote compartment in slow equilibrium	10	Estimated
C	Rate of appearance of exogenous glucose following a meal	80	Estimated
H	Intraveaneous insulin delivery rate	150	Brian RH (2001)
v_1	Fractional transfer rate	0.5	Brian RH (2001)
v_2	Fractional transfer rate	0.7	Brian RH (2001)
v_3	Fractional transfer rate	2.570	Brian RH (2001)
ϵ_1	Fractional transfer rate	0.021	Brian RH (2001)
ϵ_2	Fractional transfer rate	0.042	Brian RH (2001)
ϵ_3	Fractional transfer rate	0.435	Brian RH (2001)
ϵ_4	Fractional transfer rate	0.021	Brian RH (2001)
ϵ_5	Fractional transfer rate	0.394	Brian RH (2001)
ϵ_6	Fractional transfer rate	0.142	Brian RH (2001)
ϵ_7	Fractional transfer rate	0.47	Brian RH (2001)
n	Effect of physical exercise	0.8	Estimated
a_1	Represents pulsatile input function	$\cos 24$	Estimated
r	Stoichiometric kinetic constant involved in the production of glucose	0.25	Frank N. (2015)
γ_1, γ_2	Stoichiometric kinetic constant involved in the action of insulin on glucose	0.5, 0.15	Frank N. (2015) Frank N. (2015)
c_1, c_2	Catabolic degradation pertaining to exponential decay kinetics	0.85 0.0001	Frank N. (2015) Frank N. (2015)
l_1	Linear decay kinetics rate	10.5	Frank N. (2015)
l_2	Linear decay kinetics rate	0.0025,	Frank N. (2015)
l_3	Linear decay kinetics rate	0.0025	Frank N. (2015)
l_4	Linear decay kinetics rate	0.0025	Frank N. (2015)
j_1	Concentration of glucose in the blood plasma	10	Estimated
j_2	Concentration of glycogen in the blood plasma	40	Estimated
j_3	Concentration of glucagon in the blood plasma	50	Estimated
j_4	Concentration of insulin in the blood plasma	10	Estimated

The effect of physical exercise is calculated by finding the proportion of heartbeat for a given

minute. Given the maximum heartbeat of 175 beats per minute. Table 1 describes the variable and parameters used in the model equation.

3.1 Simulations

The authors used exercise as the only tool to check the glucose dynamics of a type 2 diabetic patient. Simulations were performed to find the level of exercise and the length of time in which glucose levels in a diabetic patient could be stabilized. Our developed glucose model is as shown below.

$$\frac{dG}{dt} = -(v_1 + nv_2Z(t))G(t)a_1 + \beta(t) + v_3 + C + rj_2j_3 - \gamma_1j_1j_4 - c_1j_1 - l_1$$

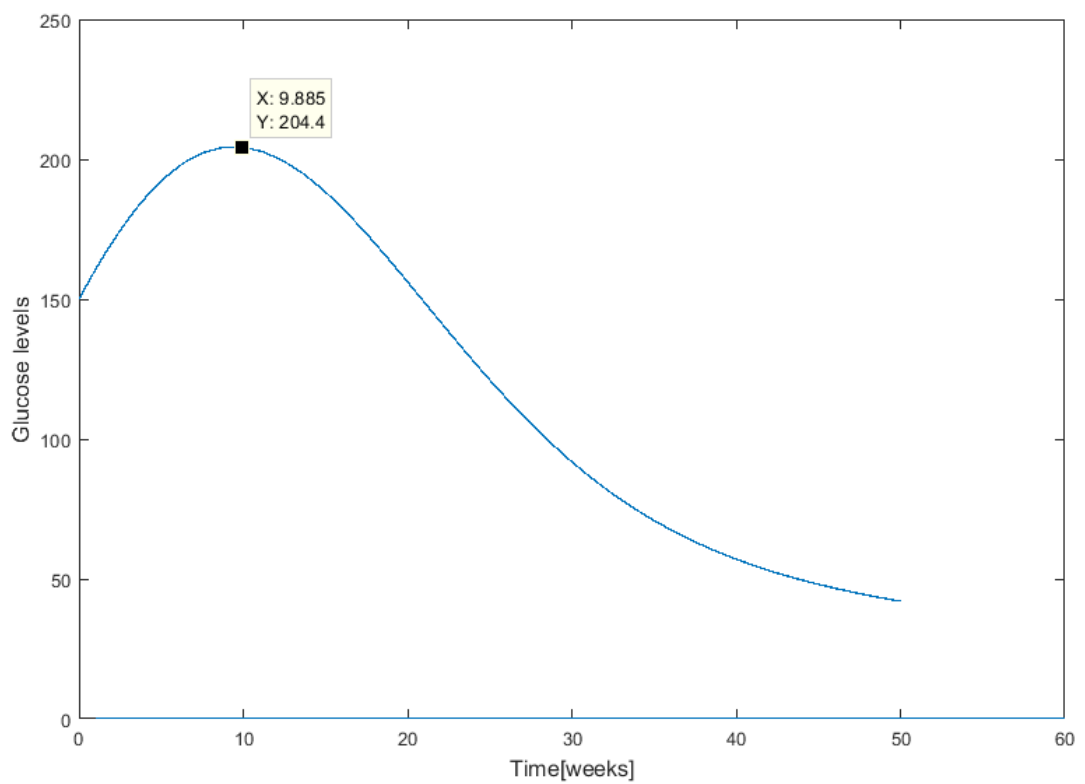


Figure 3: Plot of glucose when exercise rate is 0.35

Figure 3 shows the rise of glucose levels when exercise rate is 0.35. Glucose level rises to 204.4mg/dL which can lead to complications making the condition of the diabetic patient unstable.

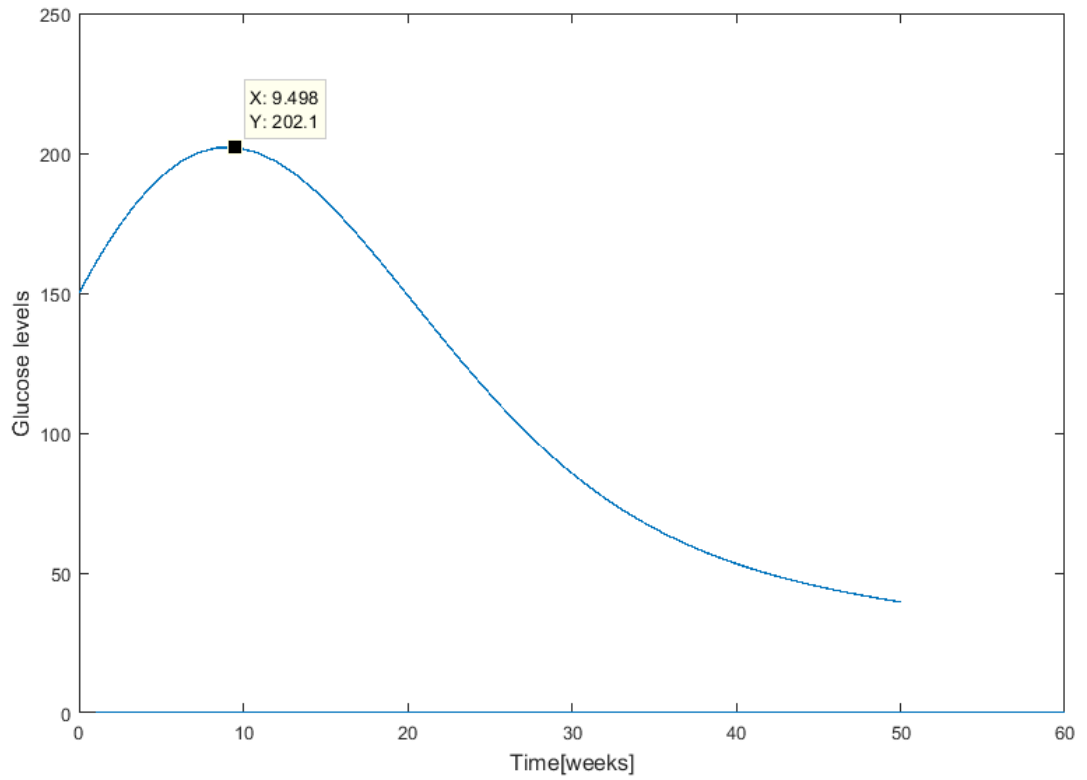


Figure 4: Plot of glucose when exercise rate is 0.37

Figure 4 shows the rise of glucose levels when exercise rate is 0.37. Glucose level rises to 201.9mg/dL which can lead to complications making the condition of the diabetic patient unstable.

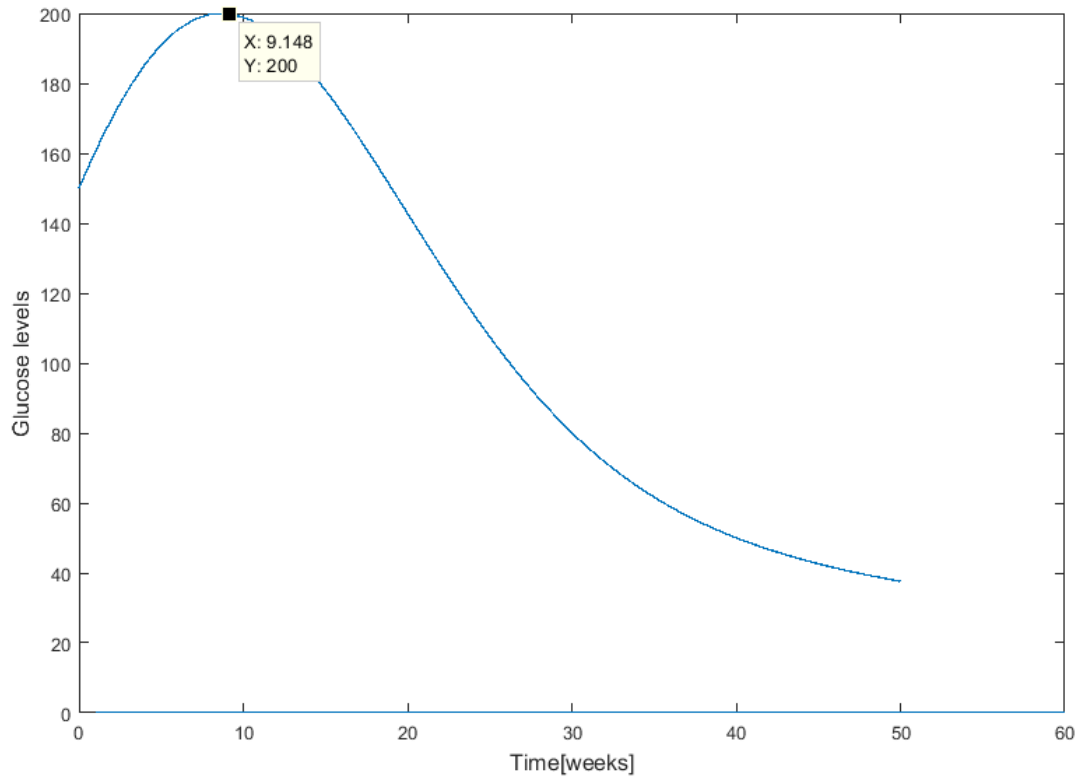


Figure 5: Plot of glucose when exercise rate is 0.39

Figure 5 shows the rise of glucose levels when exercise rate is 0.39. Glucose level rises to 200mg/dL which can lead to complications making the condition of the diabetic patient unstable.

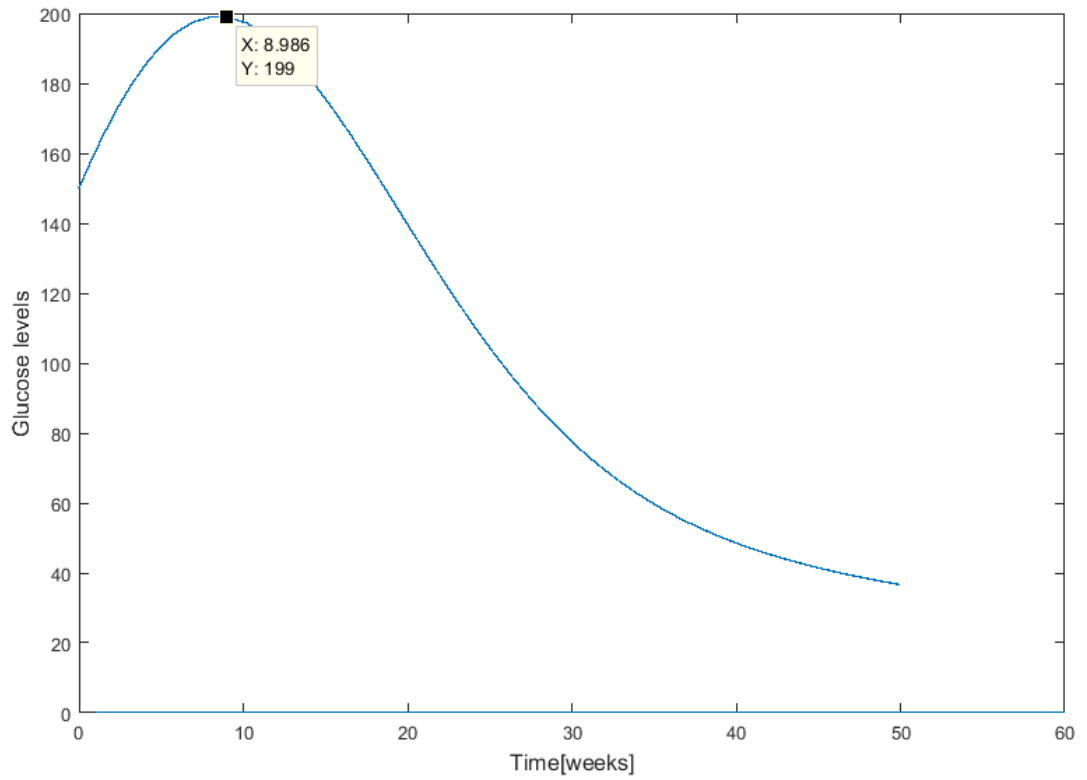


Figure 6: Plot of glucose when exercise rate is 0.4

Figure 6 shows the graph of glucose level when exercise rate is 0.4. In this graph glucose level rises to 199mg/dL in 8 to 9 weeks and begins to drop. The glucose level becomes normal in 22 to 25 weeks.

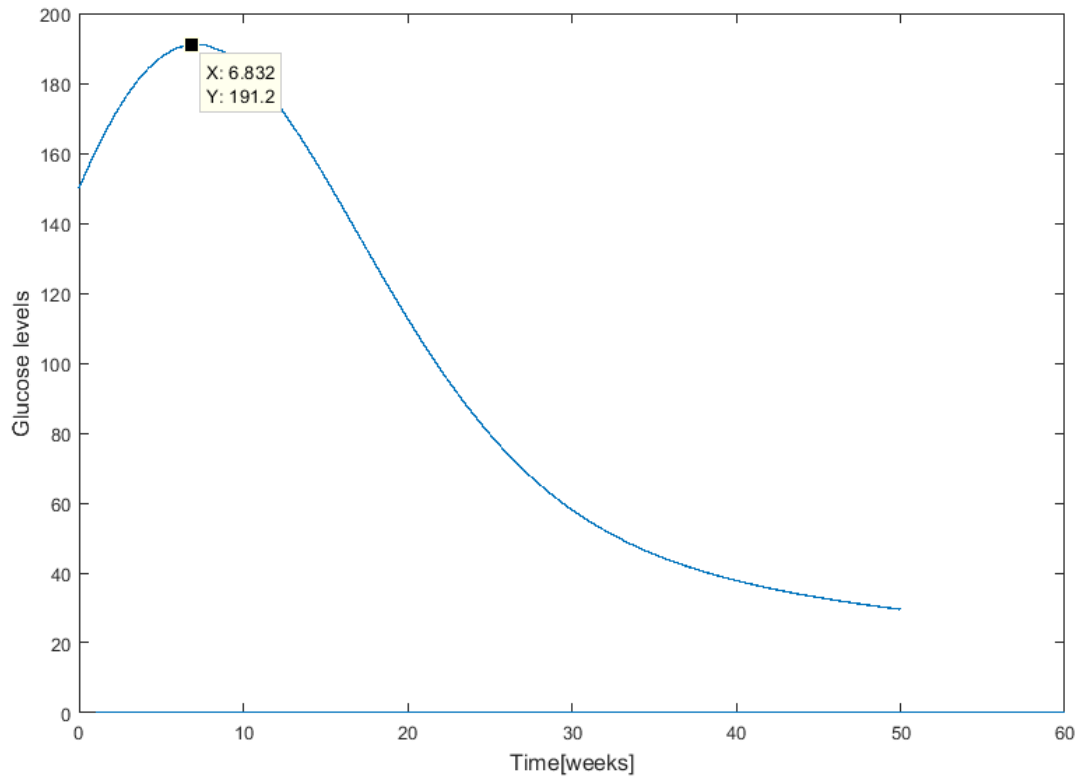


Figure 7: Plot of glucose when exercise rate is 0.5

Figure 7 shows the graph of glucose level when exercise rate is 0.5. In this graph glucose level rises to 191.2mg/dL in 6 to 7 weeks and begins to drop. The glucose level becomes normal in 20 to 22 weeks.

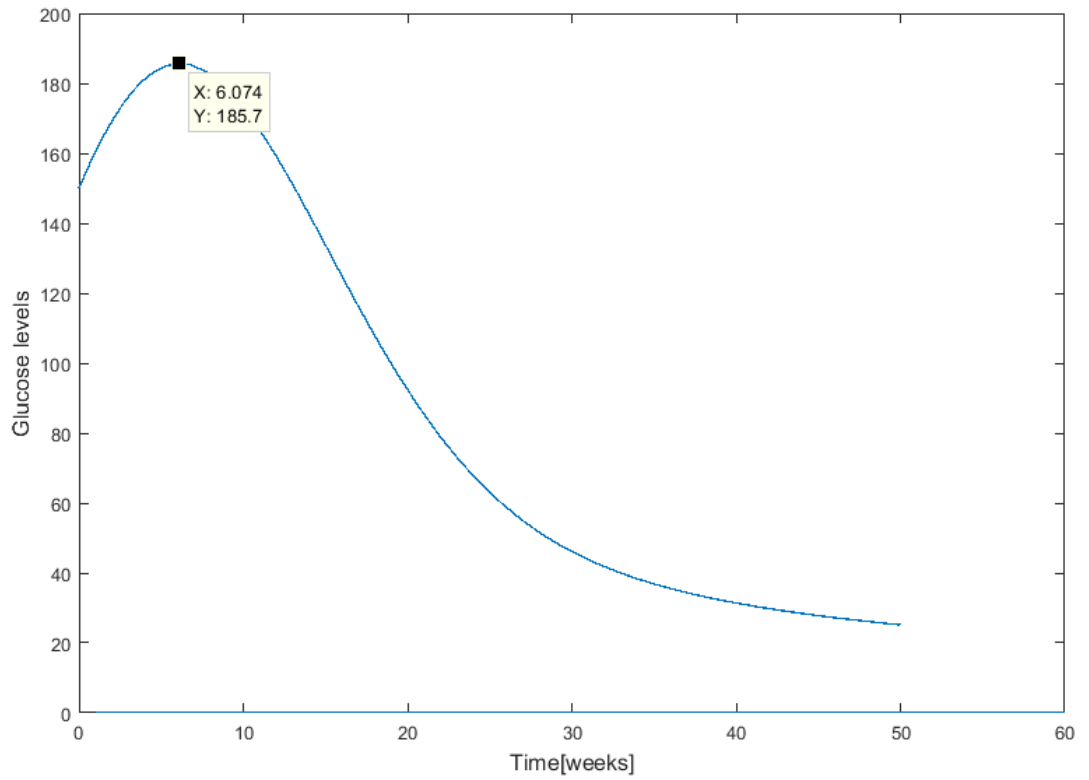


Figure 8: Plot of glucose when exercise rate is 0.6

Figure 8 shows the graph of glucose levels when exercise rate is 0.6. In this graph glucose level rises to 185.7mg/dL in 6 weeks and begins to drop. The glucose level becomes normal in 18 to 20 weeks.

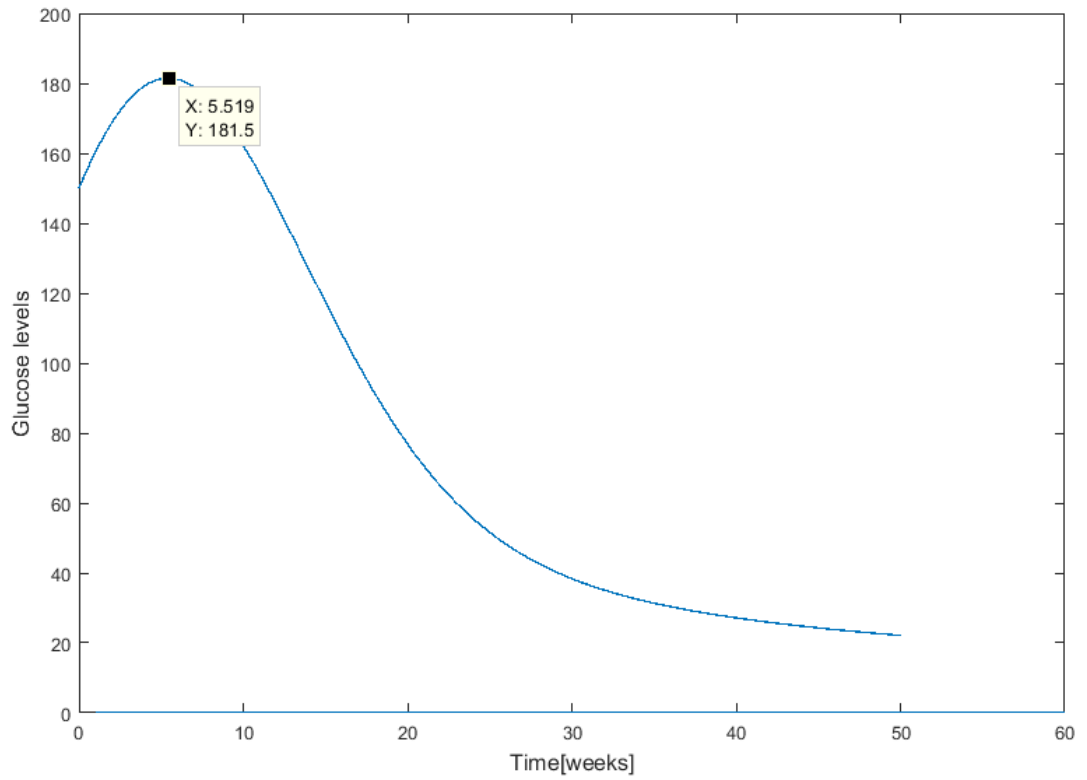


Figure 9: Plot of glucose when exercise rate is 0.7

Figure 9 shows the graph of glucose level when exercise rate is 0.7. In this graph glucose level rises to 181.5mg/dL in 5 weeks and begins to drop. The glucose level becomes normal between 16 to 19 weeks.

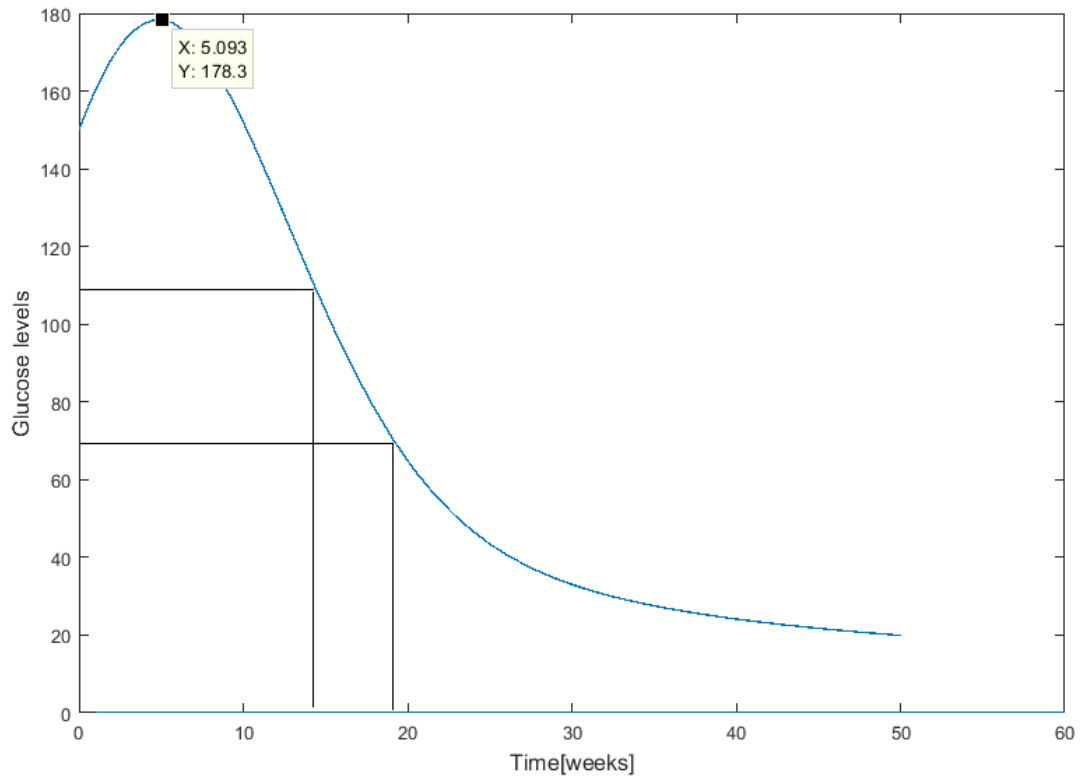


Figure 10: Plot of glucose when exercise rate is 0.8

Figure 10 shows the graph of glucose level when exercise rate is 0.8. In this graph glucose level rises to 178.3mg/dL in the first 4 to 5 weeks of exercise and begins to drop. The glucose level becomes normal between 14 to 20 weeks.

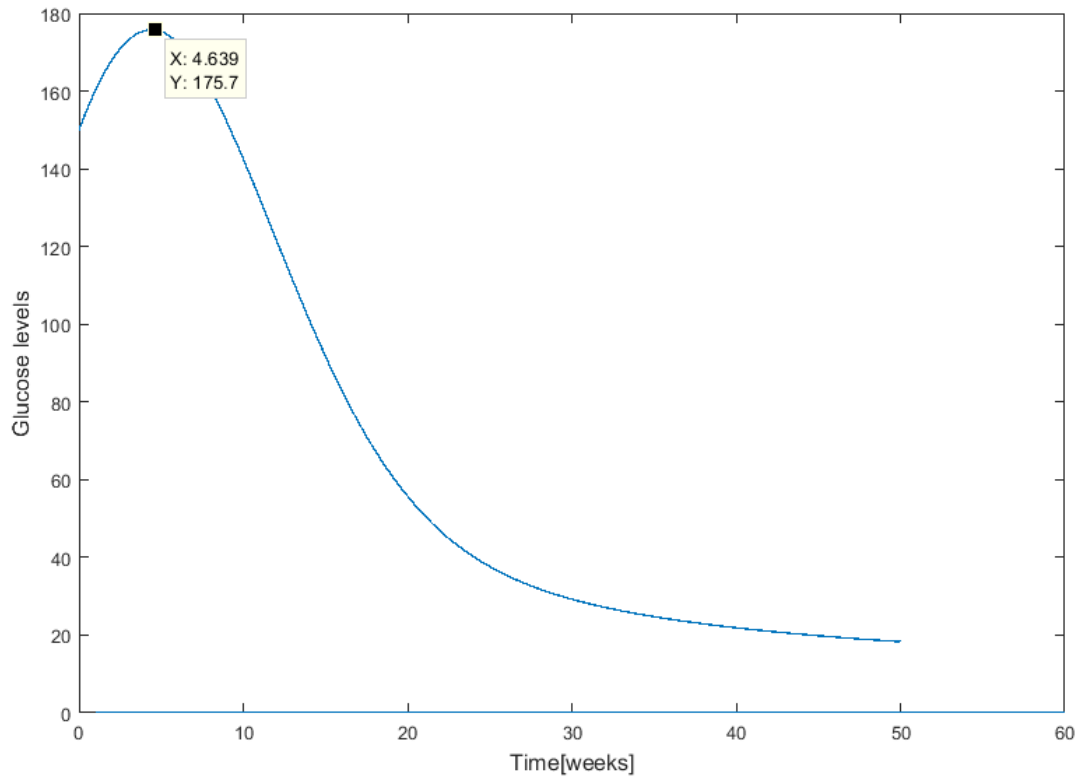


Figure 11: Plot of glucose when exercise rate is 0.9

Figure 11 shows the graph of glucose level when exercise rate is 0.9. In this graph glucose level rises to 175.7mg/dL in 4 to 5 weeks of exercise and begins to drop. The glucose level becomes normal between 12 to 16 weeks.

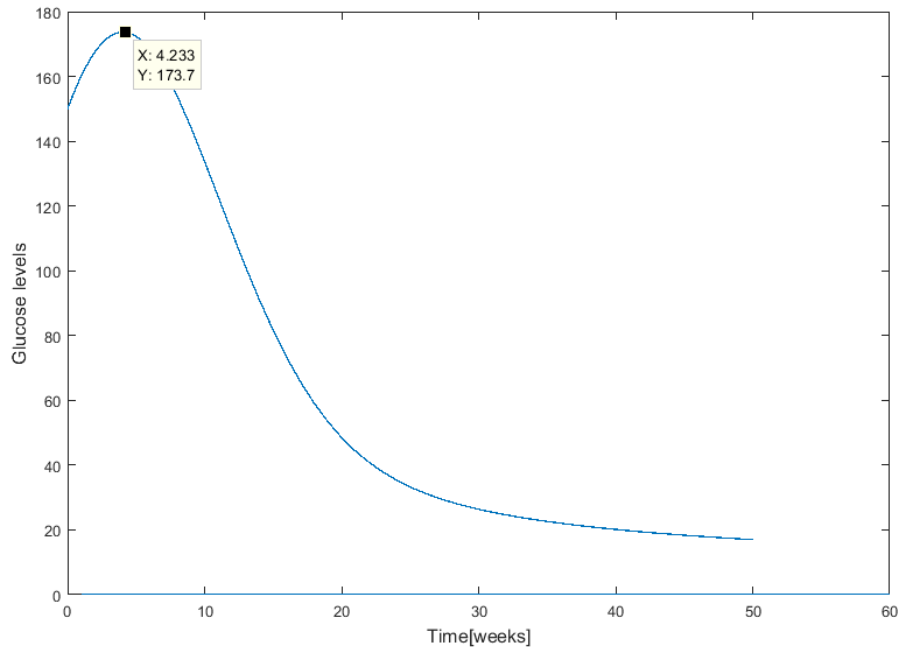


Figure 12: Plot of glucose when exercise rate is 1

Figure 12 shows the graph of glucose levels when exercise rate is 1. In this graph glucose level rises to 173.7mg/dL in 4 weeks of exercise and begins to drop. The glucose level become normal between 11 to 13 weeks.

4 Results and Discussion

Figure 3 to figure 5 shows that the conditions of a diabetic patient will not be stable due to high level of glucose. Figure 6 shows the rise of glucose levels to 199mg/dL and it begins to drop. This occurs at the end of the 7th week, the type 2 diabetic patient will have a normal glucose level at the end of the 30th week, Figure 7 shows the rise of glucose levels to 191.2mg/dL and it begins to drop. This occurs at the beginning of the 5th week, the type 2 diabetic patient will have a normal glucose level at the end of the 25th week, when the rate of exercise is 0.6 glucose level rises to 185.7mg/dL in 9 weeks and begins to fall figure 12. Glucose levels become normal at the end 30 weeks. Figure 9 shows the rise of glucose levels to 181.5mg/dL and begins to drop. This occurs in the middle of the 6th week, the type 2 diabetic patient will have a normal glucose level at the end of the 22nd week. When the rate of exercise is 0.8 from figure 10 glucose level rise to 178.3 in the middle of the 4th week and starts to fall. This is the optimal rate of exercise because the benefit of exercise last for a long period of time. Figure 11 shows the rise of glucose levels to 175.7mg/dL and begins to drop. This occurs in the middle of the 4th week, the type 2 diabetic patient will have a normal glucose level at the end of the 18th week. In figure 12 glucose level rises to 173.7 in the beginning of the 4th week and starts to fall. Glucose level becomes normal before the 20th week.

5 CONCLUSION

From the computer simulation of this paper, when the exercise was present in the model it shows that glucose level rises to a point and falls back to normal making the condition of type 2 diabetes stable. This can be seen in figure 6 to figure 12. When exercise was taken out of the model, glucose level rises very high and resulting in complications making the diabetic condition unstable. When the exercise rate was 0.8, glucose level was optimal from figure 10. Type 2 diabetes patients should try and exercise with a rate of 0.8 to get an optimal glucose level as seen from figure 10.

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