Numerical Optics Soliton Solution of the Nonlinear Schrödinger Equation Using the Laplace and the Modified Laplace Decomposition method

ABSTRACT:

In this paper, the Laplace decomposition method (LDM) and some modification, namely the Modified Laplace decomposition method (MLDM), are adopted to numerically investigate the optic soliton solution of the nonlinear complex Schrödinger equation (NLSE). The obtained results demonstrate the reliability and the efficiency of the considered method to numerically approximate initial value problems (IVPs).

Keywords: Nonlinear Schrödinger equation, Laplace Transform, Adomian polynomials.

AMS 2010 Classification:35Q40, 35Q60, 35C08, 44A10, 49M27

1. Introduction

The nonlinear complex Schrödinger equation (NLSE) is an equation which models many physical phenomena such as nonlinear optics, water waves, plasma physics, … etc . Particularly, the nonlinearity effects in an optic fiber including four-wave mixing, self-phase modulation, second harmonic generation, … etc. are modeled by the NLSE [1], [2]. Moreover, the evolution of the envelope of modulated nonlinear water wave groups are essentially described by the NLSE. All these mentioned physical phenomena are eventually interpreted by the exact solutions for specified values of the NLSE's parameters. In this paper we consider the Nonlinear Schrödinger equation (NLSE) of the form:

$$i\frac{\partial \Psi}{\partial t} + P\frac{\partial^2 \Psi}{\partial x^2} + Q\Psi\left|\Psi\right|^2 = 0 \tag{1}$$

where $\Psi(x,t)$ is a complex-valued function of real variables (x,t), and P, Q are nonzero real parameters. The NLSE (1) admits the optic soliton solution [3]:

$$\Psi(x,t) = \sqrt{\frac{-2P\alpha^2}{Q}}\tanh\left(\alpha x - 2k\alpha t + \xi_0\right)e^{i\left(kx - P\left(2\alpha^2 + k^2\right)t + \eta_0\right)} \tag{2}$$

in which $PQ < 0$ gives the de-focusing case, α is a soliton velocity, $P(2\alpha^2 + k^2)$ is a soliton wave number, k is a nonlinear frequency shift and ξ_0, η_0 are arbitrary constants.

Last recent decades, the methods of decomposing have emerged as a powerful technique and as a subject of extensive analytical and numerical studies for large and general class of linear and nonlinear ordinary differential equations (ODE's) as well as partial differential equations (PDE's), fractional differential equations, algebraic, integro-differential, differential-delay equations [4]–[17]. More precisely, the Adomian decomposition method is knowingly efficient in solving initial-value or boundary value problems without unphysical restrictive assumptions such as linearization, perturbation and so forth. The method provides the solution in an infinite series that is proven to converge rapidly with elegant computable components [4], [5], [10]. In recent years a large amount of research work concerning the developing of the ADM is investigated see for instance [18]–[23]. In particular, the modification that was proposed by Wazwaz and El-Sayed [22], suggests that the zeroth component of the decomposition series can be divided into two functions in which the first part is only assigned to the zeroth component whereas the second part is combined with recursive relation. This modified form is adopted along with the LDM to formulate the MLDM [24], [25] and to be implemented in the current numerical study.

Laplace Decomposition Method (LDM) was introduced by Khuri [11], [12] and has been successfully utilized for obtaining solutions of differential equations [6], [7], [9], [14], [17], [26]–[34] and the NLSE of our interest. As, for instance, a recent study by Gaxiola [26] who applied the Laplace-Adomian decomposition method to a NLS-like equation, namely the Kundu-Eckhaus equation, and the accuracy as well as the efficiency of the method is proved via examples, as for the nonlinear Schrodinger equation with harmonic oscillator the method of Laplace-Adomian was utilized in a comparison with another semi-analytical method to obtain approximate analytical solutions by Jaradat et. al. [28] . The Powerfulness of this method is its consistency of Laplace transform and Adomian polynomials which guarantees an accelerative, rapid convergence of series solutions when compared with the ADM itself and therefore provides major progress [11], [35], [36]. The main numerical approach in this article is implementing the Laplace decomposition method (LDM) and the Modified Laplace Decomposition method (MLDM) to the NLSE (1), for this purpose the paper is organized to fully analyze the considered methods in Section 2. Numerical results are obtained and plentifully discussed via tables, illustrations and concluding remarks in Section 3. Finally, in Section 4 a brief conclusion is given.

2. Methodology of the used Methods

2.1 LDM algorithm of the NLSE

In this section we begin with reducing the nonlinear Schrödinger equation (NLSE) (1) into a system of coupled nonlinear equations involving the real and imaginary parts, by introducing the following transformation [11], [37] :

$$\Psi(x,t) = \psi_1(x,t) + i\psi_2(x,t) \tag{3}$$

where $\psi_1(x,t)$ a and $\psi_2(x,t)$ are real –valued functions. Substituting (3) into (1) we obtain the following system of coupled real equations with an initial value problem (IVP), to take the following from:

$$\begin{aligned} &\frac{\partial \psi_1}{\partial t} + P\frac{\partial^2 \psi_2}{\partial x^2} + Q\left(\psi_1^2 + \psi_2^2\right)\psi_2 = 0 \\ &\frac{\partial \psi_2}{\partial t} - P\frac{\partial^2 \psi_1}{\partial x^2} - Q\left(\psi_1^2 + \psi_2^2\right)\psi_1 = 0 \end{aligned} \tag{4}$$

$$\begin{cases} \psi_1(x,0) = F(x) \\ \psi_2(x,0) = G(x) \end{cases} \tag{5}$$

Rewriting (4) in the following operator form:

$$\begin{aligned} L_t\psi_1 &= -\left(PL_{xx}\psi_2 + QN\left(\psi_1,\psi_2\right)\right) \\ L_t\psi_2 &= PL_{xx}\psi_1 + QM\left(\psi_1,\psi_2\right) \end{aligned} \tag{6}$$

where $L_t \equiv \frac{\partial}{\partial t}, L_{xx} \equiv \frac{\partial^2}{\partial x^2}$ are the linear differential operators, and $N\left(\psi_1,\psi_2\right) = \left(\psi_1^2 + \psi_2^2\right)\psi_2$, $M\left(\psi_1,\psi_2\right) = \left(\psi_1^2 + \psi_2^2\right)\psi_1$ symbolize the nonlinear operators.

Applying the Laplace Transform on both sides of the system in (6), and using the Laplace properties with the initial conditions, we get:

$$\begin{aligned}\psi_1(x,s) &= \frac{1}{s}F(x) - \frac{1}{s}\left[P\mathbf{L}\left[L_{xx}\psi_2\right] + Q\mathbf{L}\left[N\left(\psi_1,\psi_2\right)\right]\right] \\ \psi_2(x,s) &= \frac{1}{s}G(x) + \frac{1}{s}\left[P\mathbf{L}\left[L_{xx}\psi_1\right] + Q\mathbf{L}\left[M\left(\psi_1,\psi_2\right)\right]\right]\end{aligned} \tag{7}$$

The method assumes that the unknown functions $\psi_1(x,s), \psi_2(x,s)$ are expressed as infinite series in the form:

$$\psi_1(x,s) = \sum_{n=0}^{\infty}\psi_{1,n}(x,s) \qquad ,\psi_2(x,t) = \sum_{n=0}^{\infty}\psi_{2,n}(x,s) \tag{8}$$

And the nonlinear operators are expressed in terms of an infinite series of the well-known Adomian polynomials (see for example [4], [38]) given by:

$$\begin{aligned}N(\psi_1,\psi_2) &= \sum_{n=0}^{\infty}A_n = \sum_{n=0}^{\infty}\left(\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i\psi_{1i},\sum_{i=0}^{\infty}\lambda^i\psi_{2i}\right)\right]\right) \\ M(\psi_1,\psi_2) &= \sum_{n=0}^{\infty}B_n = \sum_{n=0}^{\infty}\left(\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[M\left(\sum_{i=0}^{\infty}\lambda^i\psi_{1i},\sum_{i=0}^{\infty}\lambda^i\psi_{2i}\right)\right]\right)\end{aligned} \tag{9}$$

Listing below a few components of Adomian polynomials:

$$\begin{aligned}A_0 &= \psi_{1,0}^2\psi_{2,0} + \psi_{2,0}^3, \\ A_1 &= 2\psi_{1,0}\psi_{1,1}\psi_{2,0} + \psi_{1,0}^2\psi_{2,1} + 3\psi_{2,0}^2\psi_{2,1}, \\ A_2 &= \psi_{1,1}^2\psi_{2,0} + 2\psi_{1,0}\psi_{1,2}\psi_{2,0} + 2\psi_{1,0}\psi_{1,1}\psi_{2,1} + 3\psi_{2,0}\psi_{2,1}^2 + \psi_{1,0}^2\psi_{2,2} + 3\psi_{2,0}^2\psi_{2,2}, \\ &\vdots\end{aligned} \tag{10}$$

$$\begin{aligned}B_0 &= \psi_{1,0}^3 + \psi_{1,0}\psi_{2,0}^2, \\ B_1 &= 3\psi_{1,0}^2\psi_{1,1} + \psi_{1,1}\psi_{2,0}^2 + 2\psi_{1,0}\psi_{2,0}\psi_{2,1}, \\ B_2 &= 3\psi_{1,0}\psi_{1,1}^2 + 3\psi_{1,0}^2\psi_{1,2} + \psi_{1,2}\psi_{2,0}^2 + 2\psi_{1,1}\psi_{2,0}\psi_{2,1} + \psi_{1,0}\psi_{2,1}^2 + 2\psi_{1,0}\psi_{2,0}\psi_{2,2}, \\ &\vdots\end{aligned} \tag{11}$$

Using (8) and (9) into (7), we have:

$$\begin{aligned}\sum_{n=0}^{\infty}\psi_{1,n}(x,s) &= \frac{1}{s}F(x) - \frac{1}{s}\left[P\mathbf{L}\left[L_{xx}\sum_{n=0}^{\infty}\psi_{2,n}\right] + Q\mathbf{L}\left[\sum_{n=0}^{\infty}A_n\right]\right] \\ \sum_{n=0}^{\infty}\psi_{2,n}(x,s) &= \frac{1}{s}G(x) + \frac{1}{s}\left[P\mathbf{L}\left[L_{xx}\sum_{n=0}^{\infty}\psi_{1,n}\right] + Q\mathbf{L}\left[\sum_{n=0}^{\infty}B_n\right]\right]\end{aligned} \tag{12}$$

According to (for example [4], [11], [12]), comparing both sides of (12) by applying the inverse Laplace transform, we obtain the subsequent components to take the following recursive relation:

$$
\begin{aligned}
&\psi_{1,0}(x,0) = F(x)\\
&\psi_{2,0}(x,0) = G(x)\\
&\psi_{1,n+1}(x,t) = -\mathrm{L}^{-1}\left[\frac{1}{s}\left[P\mathbf{L}\left[L_{xx}\psi_{2,n}\right] + Q\mathbf{L}\left[A_n\right]\right]\right] \qquad (13)\\
&\psi_{2,n+1}(x,t) = \mathrm{L}^{-1}\left[\frac{1}{s}\left[P\mathbf{L}\left[L_{xx}\psi_{1,n}\right] + Q\mathbf{L}\left[B_n\right]\right]\right] \qquad ,n \geq 0
\end{aligned}
$$

Obviously, the practical solution will be the *n*-term approximations of the infinite series (8). Thus the solution of (3) is given by:

$$
\begin{aligned}
\Psi(x,t) &= \left(\psi_{1,0} + \psi_{1,1} + \psi_{1,2} + \ldots\right) + i\left(\psi_{2,0} + \psi_{2,1} + \psi_{2,2} + \ldots\right)\\
&= \Big[R\cos\left(kx+\eta_0\right)\tanh\left(\alpha x+\xi_0\right) - Rt\Big(2P\alpha\,\mathrm{sech}\left(\alpha x+\xi_0\right)^2\left(k\cos\left(kx+\eta_0\right)\right.\\
&\left.-\alpha\sin\left(kx+\eta_0\right)\tanh\left(\alpha x+\xi_0\right)\right)\\
&+\sin\left(kx+\eta_0\right)\tanh\left(\alpha x+\xi_0\right)\Big(-k^2P + QR^2\tanh\left(\alpha x+\xi_0\right)^2\Big)+\ldots\Big]\\
&+i\Big[R\sin\left(kx+\eta_0\right)\tanh\left(\alpha x+\xi_0\right) + Rt\Big(-2P\alpha\,\mathrm{sech}\left(\alpha x+\xi_0\right)^2\left(k\sin\left(kx+\eta_0\right)\right. \qquad (14)\\
&\left.+\alpha\cos\left(kx+\eta_0\right)\tanh\left(\alpha x+\xi_0\right)\right)\\
&+\cos\left(kx+\eta_0\right)\tanh\left(\alpha x+\xi_0\right)\Big(-k^2P + QR^2\tanh\left(\alpha x+\xi_0\right)^2\Big)+\ldots\Big]
\end{aligned}
$$

where $R = \sqrt{-2\dfrac{P\alpha^2}{Q}}$.

2.2 The Modified Laplace decomposition method (LDM) algorithm of the NLSE

The methodology of the LDM is implemented to the NLSE itself (1), along with Wazwaz modification [20], [21] in which the zero components are split into two parts. According to it once we rewrite (1) in operator form, we proceed as follows:

$$
\Psi(x,s) = \frac{1}{s}\Psi(x,0) + i\left(\frac{P}{s}\mathrm{L}\left[L_{xx}\Psi\right] + \frac{Q}{s}\mathrm{L}\left[N(\Psi)\right]\right) \qquad (15)
$$

Where the nonlinear operator $N(\Psi) = \Psi\left|\Psi\right|^2$ is decomposed using Adomian polynomials [38] into infinite series:

$$
N(\Psi) = \sum_{n=0}^{\infty} A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i\Psi_i\right)\right] \qquad (16)
$$

Here we may view the few first Adomian polynomials as follows:

$$A_0 = \Psi_0 \bar{\Psi}_0$$
$$A_1 = \Psi_1 \bar{\Psi}_0 + \Psi_0 \bar{\Psi}_1$$
$$A_2 = \Psi_2 \bar{\Psi}_0 + \Psi_1 \bar{\Psi}_1 + \Psi_0 \bar{\Psi}_2$$
$$A_3 = \Psi_3 \bar{\Psi}_0 + \Psi_2 \bar{\Psi}_1 + \Psi_1 \bar{\Psi}_2 + \Psi_0 \bar{\Psi}_3 \tag{17}$$
$$A_4 = \Psi_4 \bar{\Psi}_0 + \Psi_3 \bar{\Psi}_1 + \Psi_2 \bar{\Psi}_2 + \Psi_1 \bar{\Psi}_3 + \Psi_0 \bar{\Psi}_4$$

$$\vdots$$

Now, using(15), (16) and that the decomposition of the series $\Psi(x,t) = \sum_{n=0}^{\infty} \Psi_n(x,t)$, we obtain:

$$\sum_{n=0}^{\infty} \Psi_n(x,s) = \frac{1}{s}\Psi(x,0) + i\left(\frac{P}{s}\mathrm{L}\left[L_{xx}\sum_{n=0}^{\infty}\Psi_n\right] + \frac{Q}{s}\mathrm{L}\left[\sum_{n=0}^{\infty} A_n\right]\right) \tag{18}$$

Identifying the recursive relation by comparing both sides of (18), then applying the inverse Laplace transform with its properties and using the given initial conditions we get:

$$\Psi_0(x,t) = F_1(x,t)$$
$$\Psi_1(x,t) = F_2(x,t) + i\mathrm{L}^{-1}\left(\frac{P}{s}\mathrm{L}\left[L_{xx}\Psi_0\right] + \frac{Q}{s}\mathrm{L}\left[A_0\right]\right) \tag{19}$$
$$\Psi_{n+1}(x,t) = i\mathrm{L}^{-1}\left(\frac{P}{s}\mathrm{L}\left[L_{xx}\Psi_n\right] + \frac{Q}{s}\mathrm{L}\left[A_n\right]\right) \qquad ,n \ge 1$$

The approximation is successfully obtained as the truncated series decomposition is given by:

$$\begin{aligned}\Psi(x,t) &= \Psi_0(x,t) + \Psi_1(x,t) + \dots \\ &= iR\sin(kx+\eta_0)\tanh(\alpha x+\xi_0) + R\Big(2Pt\alpha\,\mathrm{sech}(\alpha x+\xi_0)^2\left(-k\cos(kx+\eta_0) + \right. \\ &\left.\alpha\sin(kx+\eta_0)\tanh(\alpha x+\xi_0)\right) \\ &+ \tanh(\alpha x+\xi_0)\left(\cos(kx+\eta_0) + k^2 Pt\sin(kx+\eta_0)\right. \\ &\left.-QR^2 t\sin(kx+\eta_0)^3\tanh(\alpha x+\xi_0)^2\right)\Big) + \dots\end{aligned} \tag{20}$$

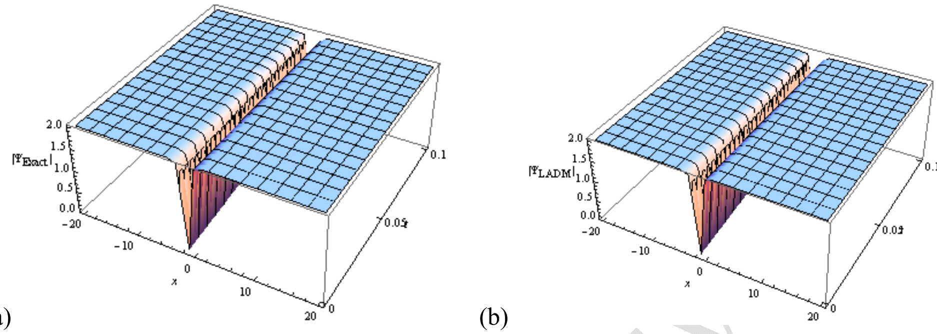
where $R = \sqrt{-2\frac{P\alpha^2}{Q}}$.

3. Numerical results and discussion

In the present numerical computations and for the numerical study purposes, we will use the 3-term approximation (14), (20), due to the massive components of the series solution . We have assumed the involved parameters are given by $k = P = 1, Q = -1, \alpha = \sqrt{2}, \xi_0 = \eta_0 = 2$, the interval of spatial coordinate x is $[-20, 20]$ and maximum value of time is taken as $t = 0.1$ sec.

3.1 LDM results

The module of the exact solution $|\Psi(x,t)|$ and the corresponding module of the numerical solution $|\Psi_{LDM}(x,t)|$ with the help of three-term approximations of the decomposition series solution are shown in **Figure 1**. Although we have used a low-order approximation which is led to high accuracy without loss of generality, this is totally achieved in **Table 1** which exhibits the absolute errors $|\Psi(x,t)-\Psi_{LDM}(x,t)|$ in constructions of the approximated $\Psi_{LDM}(x,t)$.



(a) (b)

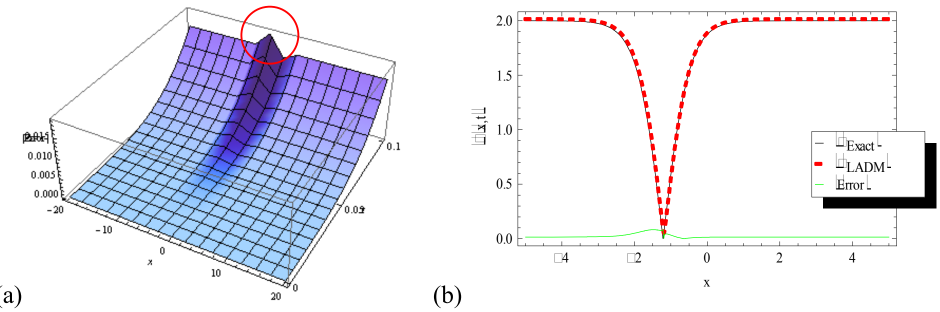
Figure 1. The plot of surface: (a) Exact module $|\Psi(x,t)|$ of the equation (2). (b) Numerical module of LDM $|\Psi_{LDM}(x,t)|$ of the equation (14).

Table 1. The Error Module $|\Psi(x,t)-\Psi_{LDM}(x,t)|$

x	t = 0.0001	0.001	0.01	0.1
-20	1.5987×10^{-14}	1.5625×10^{-10}	1.5625×10^{-6}	0.015564
-15	1.5543×10^{-14}	1.5625×10^{-10}	1.5625×10^{-6}	0.015564
-10	1.5321×10^{-14}	1.5625×10^{-10}	1.5625×10^{-6}	0.015564
-5	2.1538×10^{-14}	1.6266×10^{-10}	1.5691×10^{-6}	0.015573
0	3.0733×10^{-12}	2.9236×10^{-9}	1.4213×10^{-6}	0.01396
5	1.5321×10^{-14}	1.5625×10^{-10}	1.5625×10^{-6}	0.015564
10	1.5765×10^{-14}	1.5625×10^{-10}	1.5625×10^{-6}	0.015564
15	1.5765×10^{-14}	1.5625×10^{-10}	1.5625×10^{-6}	0.015564
20	1.5543×10^{-14}	1.5625×10^{-10}	1.5625×10^{-6}	0.015564

The calculated errors indicate a very good approximation with the actual solution by using three terms only and the error grows higher as the time value increases.

Figure 1 (a) illustrates the 3 dimensional absolute error module for values of time $[0,0.1]$ which its peak appears at $t=0.1$. Whereas, **Figure 1(b),** focuses on the peak of the surface where the absolute error module is seen at $t=0.1$ sec where it has been magnified when $x\in[-5,5]$. The exact solution $|\Psi(x,t)|$ of the equation (1), the numerical solution module $\Psi_{LDM}(x,t)$ of the equation (14) and the absolute error of the module $|\Psi(x,t)-\Psi_{LDM}(x,t)|$ are compiled in Table 2.



(a) (b)

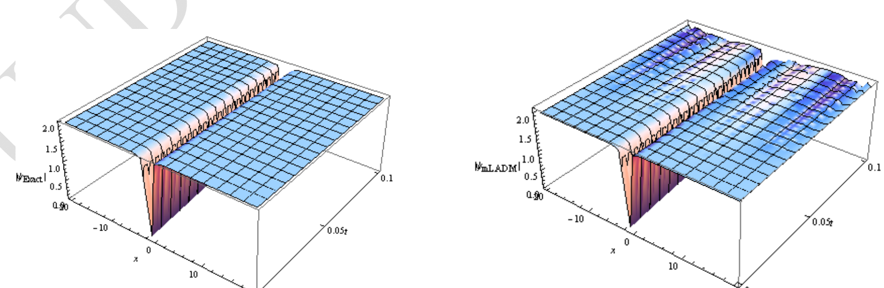
Figure 2. The plots of **: (a)** The error module $\left|\Psi(x,t)-\Psi_{LDM}(x,t)\right|$. **(b)** The peak of the Error Module Curve when $t=0.1$.

Table 2. The numerical results of the exact module (1), approximated module (14) and the module error .

x	$\left\|\Psi(x,t)\right\|$	$\left\|\Psi_{LDM}(x,t)\right\|$	$\left\|\Psi(x,t)-\Psi_{LDM}(x,t)\right\|$
-5	1.9999	2.0155	0.015573
-4	1.9985	2.0142	0.015705
-3	1.9746	1.9925	0.017932
-2	1.609	1.6589	0.049846
-1	0.58801	0.55563	0.032376
0	1.875	1.889	0.01396
1	1.9924	2.0079	0.015487
2	1.9995	2.0151	0.01556
3	2.	2.0155	0.015564
4	2.	2.0156	0.015564
5	2.	2.0156	0.015564

3.2 Modified LDM results

In this subsection the achieved approximations using modified LDM will be discussed. The interpretation of **Figure 3** indicates to the accuracy of modified LDM decreases considerably as the time interval extends which is certainly due to the complexity of the split and the massive components of the solution series (20) and the data results in **Table 3** of the absolute error $\left|\Psi(x,t)-\Psi_{mLDM}(x,t)\right|$ module prove it.



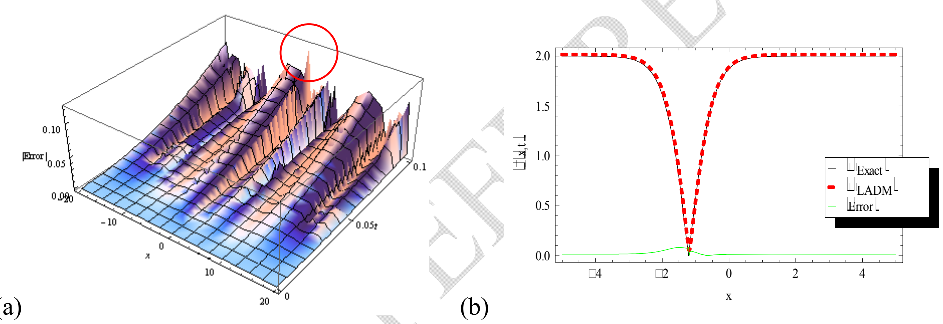
(a) (b)

Figure 3. The plot of surface of: **(a)** Exact module $\left|\Psi(x,t)\right|$ of the equation (2). **(b)** Numerical module of mLDM $\left|\Psi_{mLDM}(x,t)\right|$ of the equation (20).

Table 3. The Error Module $|\Psi(x,t)-\Psi_{mLDM}(x,t)|$

x	t = 0.0001	0.001	0.01	0.1
-20	7.9802×10^{-8}	7.9869×10^{-6}	0.00080531	0.085765
-15	5.6377×10^{-8}	5.6468×10^{-6}	0.00057374	0.066362
-10	6.6538×10^{-8}	6.6335×10^{-6}	0.00064101	0.020131
-5	1.6119×10^{-8}	1.6089×10^{-6}	0.00015788	0.012966
0	3.9583×10^{-9}	3.67×10^{-7}	8.8355×10^{-6}	0.017336
5	8.6222×10^{-8}	8.6189×10^{-6}	0.00085836	0.080951
10	7.5549×10^{-8}	7.5638×10^{-6}	0.00076522	0.084397
15	3.4809×10^{-8}	3.4498×10^{-6}	0.00031235	0.017082
20	1.0024×10^{-8}	1.0022×10^{-6}	0.00010004	0.0097977

At the end, as a completion of our numerical study a representation of the peak of the error surface at time $t=0.1$ as a two dimensional graph where the approximation and the exact solution meet and diverse.



(a) (b)

Figure 4. The plots of **: (a)** The error module $|\Psi(x,t)-\Psi_{mLDM}(x,t)|$. **(b)** The peak of the Error Module Curve when $t=0.1$.

Table 4. The numerical results of the exact module (1), approximated module (20) and the module error .

x	$\|\Psi(x,t)\|$	$\|\Psi_{mLDM}(x,t)\|$	$\|\Psi(x,t)-\Psi_{mLDM}(x,t)\|$
-5	1.9999	2.0129	0.012966
-4	1.9985	2.0526	0.05414
-3	1.9746	2.0165	0.04196
-2	1.609	1.7139	0.10489
-1	0.58801	0.79691	0.2089
0	1.875	1.8577	0.017336
1	1.9924	2.0136	0.021197
2	1.9995	2.0853	0.085782
3	2.	1.9495	0.050521
4	2.	2.04	0.039992
5	2.	2.081	0.080951

By summarizing up the numerical results of the exact module (1) from **Table 2** and **Table 4** into the table below. It is concluded that both methods are efficiently applicable to the NLS equation. However, when it comes to the issues of accuracy and convergence, the method of LDM still has the advantage of being a stabilized accurate method over the MLDM within the same number of iterations as well as the series terms of solution. Additionally, the MLDM needs a good guess of the zeroth component of the function's split as it is stated in ***Remark***.

Table 5. The module errors of the approimated solution of LDM (14) and MLDM (20).

x	$\left\|\Psi(x,t)-\Psi_{LDM}(x,t)\right\|$	$\left\|\Psi(x,t)-\Psi_{mLDM}(x,t)\right\|$
-5	0.015573	0.012966
-4	0.015705	0.05414
-3	0.017932	0.04196
-2	0.049846	0.10489
-1	0.032376	0.2089
0	0.01396	0.017336
1	0.015487	0.021197
2	0.01556	0.085782
3	0.015564	0.050521
4	0.015564	0.039992
5	0.015564	0.080951

Remark:

Obviously, any good numerical schemes should have satisfactory long time numerical behavior which is mostly accomplished by increasing the number of iterations which may be costly in time or try different split in the modified LDM. Despite some studies (see for example [24], [25]) have proposed different splits, more components of the decomposition series have to be calculated.

4. Conclusion

In this work, the LDM and modified version of it, namely the MLDM, have been successfully implemented to approximate an optic soliton solution of the nonlinear complex Schrödinger equation (NLSE) with an initial value problem (IVP). A transformation has been presented so that a system of coupled real partial differential equations is obtained and to be numerically solved in order to approximate the NLSE solution. In spite of, some studies [26], [28] in which the Laplace transform has been applied directly to the equation of interest .On the other hand, based on Wazwaz's modification [20] the solution of the NLSE is examined. The obtained results are investigated via illustrations and tables. Therefore, it is predictable, that the LDM and the MLDM are effective techniques to investigate numerical solutions of nonlinear complex problems. The LDM has the advantage of being a stabilized accurate method over the MLDM. Additionally, the considered methods are converging very rapidly with fewer terms of the series solution.

5. References

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