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Bivariate Copula based Models for the dependence of Maternal Mortality Ratio(MMR) on GDP and TFR

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Abstract

Global progress towards reducing high Maternal Mortality Rates (MMR) turned to be defeated by high maternal mortalities originating from developing countries. In rural Ghana, the lack of logistics, medical and laboratory equipment are among other key factors responsible for the high MMR despite several interventions structured to curb this menace. Improvement in the countrys health care delivery will require substantial investment into maternal and child health especially, in order to meet the national SDG target on MMR. This paper demonstrates societal benefits of investment in maternal and child health in order to stimulate stakeholders interest in resource mobilization by the correlation of MMR with such economic and demographic indicators as Gross Domestic Product (GDP) and Total Fertility Rate (TFR). The underlying probability distributions for MMR, TFR and GDP were determined using the corrected Akaike Information Criteria (cAIC) with parameters estimated via the maximum likelihood framework. MMR and TFR showed a positive association (0.83) whilst an inverse relation exist between MMR and GDP (-0.67) and TFR and GDP (-0.76). The contour and joint density plots from appendix A and B indicate a strong lower tail dependence for the bivariate Frank copula with Gamma and Lognormal margins whereas the Gumbel copula with Gamma and Lognormal margins shows strong upper tail dependence. Correlation figures tend to suggests that improved GDP as a consequence of improved socio-economic conditions of a Ghanaian mother tend to reduce Maternal Mortalities whilst increased fertility rates turn to increase MMR. Generally, evidence has been drawn to improvement in GDP.

Keywords: Maternal Mortality Ratio; Total Fertility Rate; Gross Domestic Product, Copula; Aikaike Information Criteria(AIC)

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

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1 Introduction

Global statistics indicate that there has been a significant reduction in Maternal Mortality Rate (MMR) from an estimated MMR of 385 in 1990 to an estimated 216 maternal deaths per 100 000 live births in 2015 representing an approximate reduction of 44%. There have also been records of reduced annual counts on maternal deaths from approximately 5.32 million to an estimated 3.03 million (43% decrease) with an approximate global lifetime risk of a maternal death falling considerably from 1 in 73 to 1 in 180 in the same period WHOUNICEF (2015). Although this progress is quite remarkable, global progress is reversed by high levels of MMR still recorded in developing nations. As stated by same, developing countries account for approximately 99% (3.02 million deaths) of the global maternal deaths in 2015, with sub-Saharan Africa alone accounting for roughly 66% (2.01 million). A country-wise comparison indicated that Nigeria and India were estimated to account for over one third of all maternal deaths worldwide in 2015, with an approximate 58,000 maternal deaths (19%) and 45,000 maternal deaths (15 %), respectively. Eighteen (18) other countries in sub-Saharan Africa were estimated to have very high MMR in 2015 ranging from 500 upwards to 999 deaths per 100,000 live births. Although, Ghana was not listed as part of these 18 countries, a national MMR of 350 deaths per 100,000 live births was too high for Ghana to have achieved the MDG 5 target of 185 per 100,000 live births by 2015 (**?**).

In sub-Saharan Africa, some notable causes associated with MMR are HIV/AIDS, hemorrhage, sepsis and eclampsia. Other socioeconomic and intermediate factors include inadequate investment in health care systems and population growth characterized by high fertility rates (see Jamison (2006)). For Ghana, most maternal deaths are preventable with postpartum haemorrhage, hypertensive disorders, abortion and sepsis contributing about 65% of all causes. It is believed such deaths could have been averted by more than half if accelerated investment were made into providing access to essential reproductive health services such as family planning, skilled attendants, administration of oxytocin and misoprostol for management of postpartum haemorrhage, and magnesium sulphate for treatment of pregnancy-induced hypertensive disorders . In rural Ghana in particular, the lack of logistics, medical and laboratory equipment are identified as factors responsible for the teaming number of maternal deaths in rural Ghana despite several interventions towards reducing this menace Apanga (2018). Especially, in this new era of Sustainable Development Goals (SDGs), if proper interventions are not taken, it will push the national goal of zeroing maternal deaths down to an unwanted direction.

While global commitment to address this ”Canker” was fashioned as improving infants and maternal health in the Millennium Development Goals 4 and 5 and now SDG 3 to reduce the global MMR to less than 70 per 100,000 births with countries targeting a maternal mortality rate below twice the global average, improving the country's health care systems in order to achieve the SDG targets on maternal and child health and other health related targets will require additional investments. To demonstrate the accrued societal benefits of investing into maternal and child health is crucial in stimulating stakeholders interest in resource mobilization Amiri (2013).

According to Jamison (2006), the root causes and correlates of maternal mortality must be understood in attempt to address the high MMR we are confronted with. Consequently, we seek to understand how MMR relates TFR and GDP in Ghana.

Definitions

Maternal mortality is defined as deaths occurring in women, while pregnant or within 42 days of termination of pregnancy irrespective of the duration and site of the pregnancy, from any cause related to or aggravated by the pregnancy or its management, but not from accidental or incidental causes WHO (1992).

It measured using the Maternal Mortality Ratio (MMR) which is defined as the number of maternal deaths per every 100,000 live births. That is

$$MMR = \frac{N_{MD}}{N_{LB}} \times 100000 \tag{1.1}$$

where N_{MD}: is the number of maternal deaths, N_{LB}: Number of live births.

In this paper, we seek to study how Ghana's MMR is related to the Gross Domestic Product (GDP) and the rapid population growth which is characterized by the Total Fertility Rate (TFR). According to the Population Reference Bureau, Total Fertility Rate (TFR) is the average number of children a woman would have assuming that current age-specific birth rates remain constant throughout her childbearing years. It is calculated by summing across the average number of births per woman in five-year age groups. That is

$$TFR = 5 \times \sum(ASFR) = 5 \times \left(\frac{N_{bw}[15-19]}{P_w[15-19]} + \cdots + \frac{N_{bw}[45-49]}{P_w[45-49]} \right), \tag{1.2}$$

where N_{bw}: number of births to women aged, P_w: population women aged. This means that TFR depends not only on the number of births but also on the number of women across the childbearing age groups; hence it is important to note that an increase in the number of births does not necessarily lead to an increase in TFR. The GDP, on the other hand, is the sum of consumption (C), investment (I), government spending (G) and net exports $(X-M)$ that is :

$$GDP = C + I + G + (X - M).$$

2 Method and Material

2.1 Data and Source

The Data used for this study contains MMR, TFR and GDP indicators of Ghana from 1990 to 2018 obtained on-line from the Ghana Economic OutlookF.Econs and World-BankWorldBank. The R statistical software as used for the analysis.

2.2 Definition: Copulas and Sklar's theorem

For an n-variate distribution $F \in F(F_1, \ldots, F_n)$, with ith univariate marginal F_i, the copula associated with F is a distribution function $C\colon [0,1]^n \to [0,1]$ that satisfies $F(y) = C(F_1(y_1), \ldots, F_n(y_n), y \in R^n$. If F is a continuous n-variate distribution function with univariate marginals $F_1, ..., F_n$, and quantile functions $F_1^{-1}, \ldots, F_n^{-1}$, so that $C(u_1 \ldots u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n))$ then, the function C is called a *Copula*. The theorem by SklarSklar (1959) states that for a joint distribution function F, there is a unique copula C that satisfies

$$F(y) = C(F_1(y_1), \ldots, F_d(y_d)) = P(U_1 \le F_1(y_1), \ldots, U_d \le F_d(y_d)), \quad y \in R^d$$

.

2.3 Modeling with copula

The joint CDF (Cumulative Distribution Function) of a multi-dimensional copula, $R(x,y,z)$ for the random vector (x,y,z) can be defined as

$$R(x,y,z) = C(F(x), G(y), H(z)), \quad x,y,z \in R, \tag{2.1}$$

where $F(x)$,$G(y)$, $H(z)$ are marginal distributions and $C\colon [0,1]^3 \rightarrow [0,1]$. The Sklar theorem suggest that, with $R(x,y,z)$ known, $C, F(x), G(y), H(z)$ can be uniquely determined. As a direct consequence of equation (1), a model for (X,Y,Z) can be structured with

$$C \in C(\theta), \quad F \in F(\sigma), \quad G \in G(\omega), \quad H \in H(\tau)$$

selected from a known parametric family. Needless to say that the choice of an appropriate copula, therefore, is very crucial and achievable through the so-called Frechet-Hoeffding bounds. The joint cdf of the n-variate cdf with uniform marginals is bounded both below and above by Frechet-Hoeffding bounds F_L and F_U defined as

$$F_L(y_1,\ldots,y_n) = max\left[\sum_{i=1}^{n} U_i - (n-1), 0\right] = W \tag{2.2}$$

and

$$F_U(y_1,\ldots,y_n) = min(F_1(y_1),\ldots,F_n(y_n)) = M \tag{2.3}$$

so that

$$max\left[\sum_{i=1}^{n} U_i - (n-1), 0\right] \leq (F_1(y_1),\ldots,F_n(y_n)) \leq min(F_1(y_1),\ldots,F_n(y_n))$$

except that for some $n > 2$, F may or may not be a cdf under certain conditions(see theorem 3.6, Joe (1997)), both F_L and F_U are cdf when $n = 2$. More generally, the copula representation for the Freechet-Hoeffding bounds is defined as

$$\begin{aligned} C_L(y_1,\ldots,y_n) &\leq C(y_1,\ldots,y_n) \leq C_U(y_1,\ldots,y_n) \\ &= W(u,v) \leq C(u,v) \leq M(u,v), \quad u,v \in [0,1], \end{aligned}$$

see Trivedi (2005).

Theorem : 1

*If C is an n-dimentional copula, then for every **u** in $[0,1]^n$,*

$$W^n(\boldsymbol{u}) \leq C(\boldsymbol{u}) \leq M^n(\boldsymbol{u})$$

In our case where $n = 3$, the Frechet-Hoeffding lower bound W^3 is not copula in the sense of the aforementioned. However, it is the best possible lower bound in this regards:

Theorem 2

*For $n > 3$ and any **u** in$[0,1]^n$, there is an n-copula C which depends on **u** such that*

$$C(\boldsymbol{u}) = W^n(\boldsymbol{u})$$

For every $n \geq 2$,$C\colon [0,1]^2 > W^2$(the Frechet-Hoeffding lower bound),and every $C\colon [0,1]^n < M^n$(the Frechet-Hoeffding upper bound). See P.Embrechts (2001).

2.4 Fitting Marginal Distributions

To select the appropriate marginal distribution, the dataset is fitted to : Exponential, Normal, Lognormal, Weibull, Pareto Gamma and Gumbel distributions and with their model parameters estimated via Maximum Likelihood Estimation(MLE). The model selection procedure is based on the Akaike Information Criterion (AIC) as discussed briefly below.

Akaike Information Criterion (AIC)

The Akaike Information Criterion (AIC)is based on Kullback-Leibler (K-L) information loss (due to Kullback and Leibler, 1951). Let f denotes full reality or truth (f is non-parametric) and let the distribution g denote an approximation to the truth f. Also let $I(f,g)$ be the information loss when model g is used to approximate f; this is defined for continuous functions as;

$$I(f,g) = \int f(y)\log\left(\frac{f(y)}{g(y \mid \theta)}\right)dy.$$

This can be expressed as the difference between statistical expectations with respect to the truth f.

$$I(f,g) = E_f[\log f(y)] - E_f[\log(g(y \mid \theta))],$$

$E_f[\log f(y)]$ is an unknown constant that depends only on the unknown true distribution so that;

$$-E_f[\log(g(y \mid \theta))] = I(f,g) - C$$

Multiplying the left hand side by -2 and rearranging yields the so-called Akaike Information Criterion.

$$AIC = -2\log(L(\hat{\theta} \mid y)) + 2K.$$

An additional bias adjustment criterion called AICc (Hurvich and Tsai, 1989 cited in Burnham (1998) Burnham and Anderson,1998) is used and defined as follow;

$$AICc = -2\log(L(\hat{\theta} \mid y)) + 2K + \frac{2K(K+1)}{n-K-1}, \tag{2.4}$$

since the data points used is < 40. To allow for quick comparison and ranking of candidate models, the $\Delta_i AICc$ is used. If $\Delta_i AICc \leq 2$, there is substantial support for making inferences with the model, for $4 \leq \Delta_i AICc \leq 7$ there is less substantial evidence and for $\Delta_i AICc > 10$ there is no substantial evidence and such models fail to explain the random variation inherent in the data. (see Burnham (1998)).

2.5 Fitting Copula Models

Since the fitting of marginal distribution herein is based on MLE, let $(x_1, \cdots, x_n)$ be some observation with $f_j(.;\theta_j)$ and $F_j(.;\theta_j)$ as the jth marginal density and distribution function respectively. Then the MLE involves maximizing the likelihood

$$L(x;\theta) = \prod_{j=1}^{d} c(F_i(x_{i,j};\beta_j);\alpha) f_i(x_{i,j};\beta_j), \quad i = 1,\dots,n,$$

where α denote the parameter of the copula C and θ is a parameter vector. The loglikelihood function is

$$l(\theta) = \sum_{i=1}^{n} \log c\bigg(F_1(x_{i,1};\beta),\dots,F_i(x_{i,d};\beta),\alpha) + \sum_{i=1}^{n}\sum_{j=1}^{d} \log f_i(x_{i,j};\beta)\bigg).$$

The Parameter estimation is based on the two-staged Inference Functions for Marginals(IFM) method Joe (1996). To be precise;

$$\hat{\alpha}_{IFM} = argmax \sum_{i=1}^{n} \log c\bigg(F_1(x_{i,1};\hat{\beta}_{IFM}),\dots,F_i(x_{i,d};\hat{\beta}_{IFM}),\alpha\bigg) \tag{2.5}$$

and

$$\beta_{IFM} = argmax \sum_{i=1}^{n} \log f_i(x_{i,j};\beta_j). \tag{2.6}$$

2.6 Dependence Measure

For the random vector (X, Y, Z), the dependence structure is obtained through pairwise comparison of the outcomes (X, Y),(X, Z) and (Y, Z). A basic measure of association between two random variables is the Pearson's correlation coefficient which is regarded as inappropriate and often misleading P.Embrechts (2001). The Spearman's rho and Kendall's tau coefficients which are the most commonly used provide the best alternative as a measure of dependence for non-elliptical distributions to which the linear correlation coefficient is inefficient.

2.6.1 Kendall's tau and Spearman's rho

A: Spearman's rho

The copula C model by itself characterize the dependence in a pair (X, Y). Suppose that a random sample pair $(X_1, Y_1), \ldots, (X_n, Y_n)$ is given from some pair (X, Y). Also let (S_i, T_i) denote the ranked pair of (X, Y). Rescaling the axes by a factor of $\frac{1}{n+1}$, we obtain a set of points in the unit square $[0, 1]^2$. The rational is to compute the correlation between the rank (S_i, T_i) via the Pearson's approach so that

$$\rho_n = \frac{\sum_{i=1}^{n}(S_i - \overline{S_i})(T_i - \overline{T_i})}{\sqrt{\sum_{i=1}^{n}(S_i - \overline{S_i})^2(T_i - \overline{T_i})^2}} \in [-1, 1],$$

where

$$\overline{S_i} = \frac{\sum_{i=1}^{n} S_i}{n} = \frac{n+1}{2} = \frac{\sum_{i=1}^{n} T_i}{n} = \overline{T_i}$$

$$\rho_n = \frac{12}{n(n+1)(n-1)} \sum_{i=1}^{n} S_i T_i - 3\frac{n+1}{n-1}$$

ρ_n shares with Pearson's classical correlation coefficient, r_n, the property that its expectation vanishes when the variables are independent. An asymptotically unbiased estimator of ρ is

$$\rho = 12 \int_{[0,1]^2} uvdC(u, v) - 3 = 12 \int_{[0,1]^2} C(u, v)dudv - 3 \tag{2.7}$$

which also takes the form

$$\rho = 12 \int_{[0,1]^2} \{C(u, v) - uv\}dudv,$$

see Genest (2007).
This can also be written as

$$\rho = 12E(UV) - 3,$$

where

$$E(UV) = \int_0^\infty \int_0^\infty UVdC(U, V)$$

, see P.Embrechts (2001).

B: Kendall's tau

The Kendell's tau just like the Spearman's rho is based on ranks. Kendall's correlation coefficient, defined as follows
Definition : *Consider two independent and identically distributed continuous bivariate random variables (X, Y) and (X^*, Y^*) with marginal distribution $F(X)$ for X and $(X^*$ and marginal distribution $F(Y)$ for Y and Y^*.The measure of association, Kendall's tau, $\tau_k(X, Y)$, is given by*

$$\tau_k(X, Y) = P[(X - X^*)(Y - Y^*) > 0] - P[(X - X^*)(Y - Y^*) < 0]$$

This can be interpreted as the difference between probability of concordance and the probability of dis-concordance between the random variables. Two pairs (u_1, u_2),$(v_1, v_2) \in [0,1]^2$ are concordant, if both components(u_1, u_2) are either both greater or both less than their respective components of the second pair, (v_1, v_2), i.e. if

$$(u_1 - v_1)(u_2, v_2) > 0$$

else, they are discordant. In copula terminology, Kendall's tau is defined as

$$\begin{aligned} \tau_k(X,Y) &= \int_0^1 \int_0^1 C(u,v)dC(u,v) - 1 \\ &= 4E[C(UV)] - 1 \end{aligned} \tag{2.8}$$

(see Frees and Valdez, 1998; Klugman et al, 2012).

2.6.2 Tail Dependence

It may interest one wanting to know for example; the probability that MMR fall below (or exceed) some level given that the TFR is also below (or exceed) another value. This conditional probability that one index is extreme given another extreme event often requires a dependence measure for upper and lower tails of the distribution. Now lets consider the random variables X and Y with marginal distributions $G(X)$and $H(Y)$. The index of upper tail dependence, I_U

$$I_U = \lim_{v\to 1} P\{X > G^{-1}(v) \mid Y > H^{-1}(v)\}$$

this is equivalent to

$$I_U = \lim_{u\to 1^P}\{G(X) > u \mid H(Y) > u\}$$

$$= \lim_{v\to 1} P\{U > v \mid V > v\} = \lim_{v\to 1} \frac{P\{U > v, V > v\}}{P(V > v)}, U, V \in [0,1]$$

$$= \lim_{v\to 1} \frac{1 - P(U \le v) - P(V \le v) + P(U \le v, V \le u)}{1 - P(V \le v)}$$

$$I_U = \lim_{v\to 1} \frac{1 - 2v + C(v,v)}{1 - v}. \tag{2.9}$$

This justifies the assertion that the copula in itself is a measure of dependence and that the tail dependency of $X$and Y can be measured via the Copula rather than their marginal distributions. The index of lower tail dependence, I_L is obtained by substituting $v = 1 - v$. so that

$$I_L = \lim_{u\to 0} \frac{C(v,v)}{v}, \quad v \in [0,1], \tag{2.10}$$

see Kullback (1951).

2.7 Goodness-of-fit tests on Copula

The goodness-of-fit tests on copula are based on empirical copula.

$$C_n(\mathbf{u}) = \sqrt{n}(C_n(\mathbf{u}) - C_{\theta_n}(\mathbf{u})), \quad \mathbf{u} \in [0,1], \tag{2.11}$$

where C_n is the empirical copula defined by

$$C_n(\mathbf{u}) = \frac{1}{n}\sum_{i=1}^{n} I(\hat{U} \le u), \quad \mathbf{u} \in [0,1]^d$$

and C_{θ_n} is an estimator of C under the null hypothesis that

$$H_0 \colon C \in \{C_\theta\}.$$

The estimator θ_n is based on ranks via the inversion Kendall's tau and Spearman's rho, or the maximum pseudo-likelihood estimator as proposed by Genest (1995).

3 Numerical Results and Discussions

3.1 Correlation Analysis

The figure 1 shows a pairwise correlation obtain via Kendall's tau along with scatter plot and probability histogram. It is observed that while all the pairs are highly correlated, both MMR and TFR are inversely proportional to GDP.

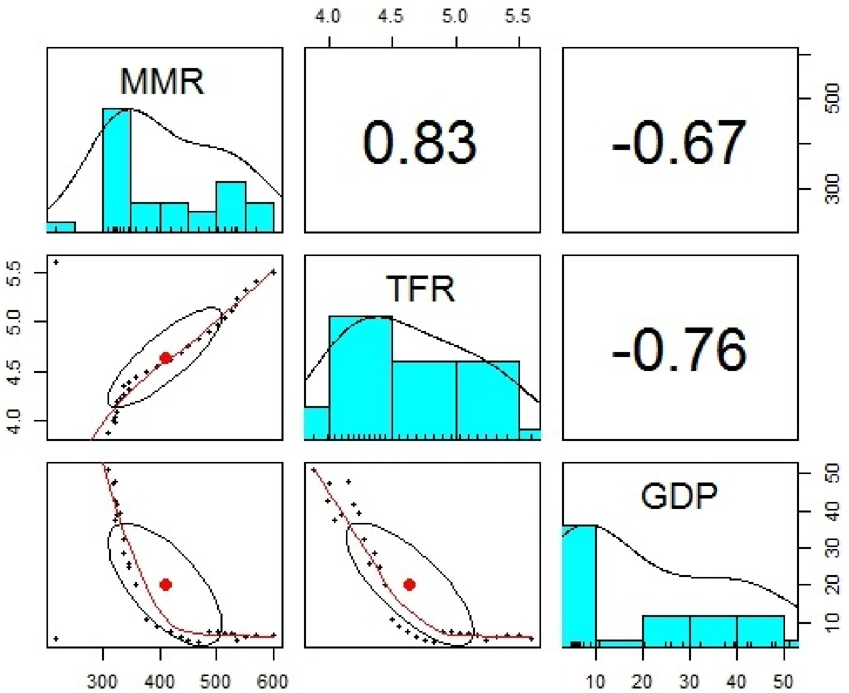

Figure 1: Correlation analysis on MMR, TFR and GDP of Ghana

3.2 Choice of Bivariate Copula Model

MMR and TFR

The table 1 shows results obtained from fitting bivariate Copula models to Maternal Mortality Rate and the Total Fertility Rate data with their standard errors in brackets.

Copula	Parameter Estimate	τ_k	I_L	I_U	AICc	Δ AICc
Gaussian	alpha =0.740	0.530	0.000	0.000	-67.90	24.74
Clayton	alpha =3.386 (0.831)	0.629	0.815	0.000	-31.600	61.04
Gumbel	alpha =4.025 (0.683)	0.752	0.000	0.812	-45.31	47.33
Frank	alpha =18.550 (3.491)	0.803	0.000	0.000	-52.95	39.69
Joe	alpha =4.957 (0.953)	0.675	0.000	0.850	-39.11	53.53
Tawn Type 2	Par 1=13.180 Par 2= 0.860	0.800	0.000	0.850	-92.64	0.000

Table 1: Results for fitting Bivariate Copula to MMR and TFR data

The copula fitting process selects the Tawn type 2 copula model as the best candidate among the 6 fitted models with a correlation value 0f 0.80 against the empirical correlation value of 0.83; explaining a strong dependence in the upper tail (i.e $I_U = 0.85$).

MMR vs GDP and TFR vs GDP

From the empirical correlation figures both MMR versus GDP and TFR verses GDP move in the opposite direction. Only the Gaussian and Frank Copulae allow for negative values of τ. The fitted results are shown in the tables 2 and 3 below.

Copula	Parameter Estimate	τ_k	I_L	I_U	AICc	Δ AICc
Gaussian	α = -0.71	-0.50	0.00	0.00	-23.19	5.58
Frank	α = -8.98 (1.965)	-0.64	0.00	0.00	-28.77	0.00

Table 2: Model Summary for MMR and GDP data.

The corresponding copula model for this data is given by

$$C(u_1, u_2) = \frac{1}{-8.98} \ln \left(1 + \frac{(e^{-8.98u_1} - 1)e^{-8.98u_2} - 1)}{e^{-8.98} - 1} \right), \quad u_1, u_2 \in [0, 1].$$

Copula	Parameter Estimate	τ_k	I_L	I_U	AICc	Δ AICc
Gaussian	alpha =-0.875	-0.67	0.00	0.00	-33.34	8.04
Frank	alpha = -13.03	-0.73	0.00	0.00	-41.38	0.00

Table 3: Table 3:Model Summary for TFR and GDP data.

Also, the copula model for this data is

$$C(u_1,u_2)=\frac{1}{-13.03}\ln\left(1+\frac{(e^{-13.03u_1}-1)e^{-13.03u_2}-1)}{e^{-13.03}-1}\right),\quad u_1,u_2\in[0,1].$$

3.3 Selecting Marginal Distributions

The following tables show the summary of results obtained by fitting the three indicators to distributions from the ”fitdistplus” and ”actuar” add on packages along with their AIC difference.

Distribution	**Parameter Estimate**	**AICc**	**ΔAICc**
Exponential	rate=0.002(0.000)	408.99	57.16
Gamma	shape=17.274(4.437) rate=0.042(0.011)	351.83	0.00
Lognormal	meanlog=5.985(0.045) sdlog=0.243 (0.032)	351.94	0.11
Gumbel	a=361.995(16.696) b=85.134(12.140)	352.48	0.65
Weibull	shape=4.575 (0.659) scale=0.329(1.000)	353.34	1.51
Normal	mean=409.345(18.251) sd=98.287(12.906)	352.86	1.03
Student t	df=0.142(0.028)	565.63	213.80

Table 4: Results for Fitting MMR in Ghana

Distribution	**Parameter Estimate**	**AICc**	**△AICc**
Exponential	rate= 0.216(0.040)	149.13	104.581
Gamma	shape=89.201(23.382) rate=19.237(5.056)	351.839	307.283
Lognormal	meanlog=1.528(0.020) sdlog=0.106(0.014)	45.06	0.520
Gumbel	a=4.399(0.081) b=0.413(0.061)	44.552	0.000
Weibull	shape=9.969(1.405) scale=4.864(0.096)	48.720	4.231
Normal	mean=4.637(0.092) sd= 0.495(0.064)	45.920	1.374
Student t	df=0.564(0.131)	242.373	197.8202

Table 5: Table 5: Summary results from Fitting TFR in Ghana.

Distribution	**Parameter Estimate**	**AICc**	**△AICc**
Exponential	rate= 0.049(0.009)	234.731	2.915
Gamma	shape=1.539(0.369) rate=0.076(0.021)5	234.14	2.334
Lognormal	meanlog= 2.651(0.160) sdlog=0.860(0.113)	231.827	0.000
Gumbel	a= 12.783(2.290) b=11.765(1.893)	242.41	10.592
Weibull	shape=1.267(0.186) scale=21.955(3.409)	234.678	2.852
Normal	mean=20.287(3.009) sd= 16.204 (2.1276)	248.306	16.485
Student t	df=0.308(0.064)	333.324	101.504

Table 6: Summary results from Fitting GDP in Ghana.

From tables 4, 5 and 6 (marginal distributions), those with $\Delta AICc < 2$ indicate a strong support for making inferences.GDP can be described by the Lognormal distribution. All distributions fitted except the exponential and student t distributions are insufficient evidence for MMR while the Gumbel, Lognormal, Gamma favors the TFR.

3.4 Fitting Copula with Continuous Marginals

Here we consider only MMR and TFR data where a positive association exists. The best-fitted copula for this data is the Tawn-Type 2 copula (which extreme value family), however since we cannot specify the dimension of this copula through the extreme value copula, we considered the Tawn, Gumbel and Frank families for situations where the margins are gamma and lognormal for MMR and TFR respectively. Figure 2 below shows the panel plots obtained from a 2000 simulated samples from the respective families and marginals;

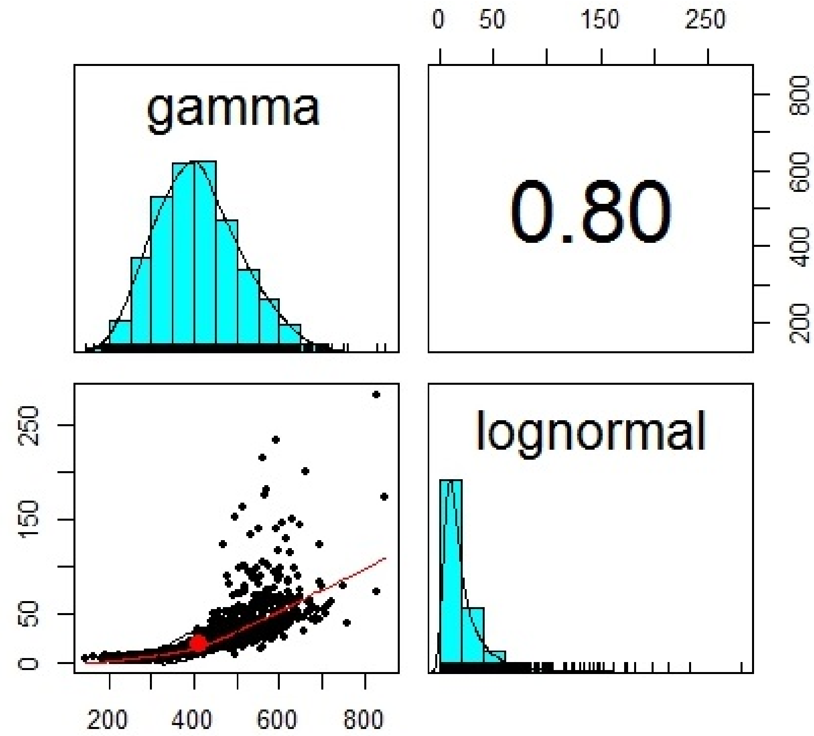

Figure 2: Panel Plot Frank Copula Gamma and Lognormal margins

$$C(x, y) = \exp -[(-\ln x)^{18.55} + (-\ln y)^{18.55}]^{0.248447}$$

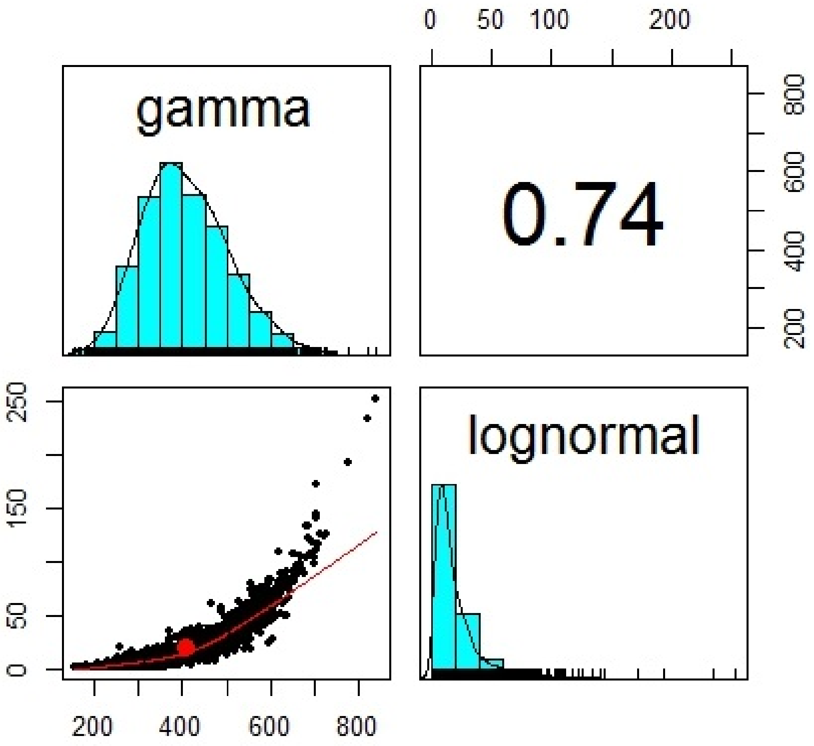

Figure 3: Panel Plot Gumbel Copula with Gamma and Lognormal margins

$$C(x, y) = \exp -[(-\ln x)^{4.025} + (-\ln y)^{4.025}]^{0.248447}$$

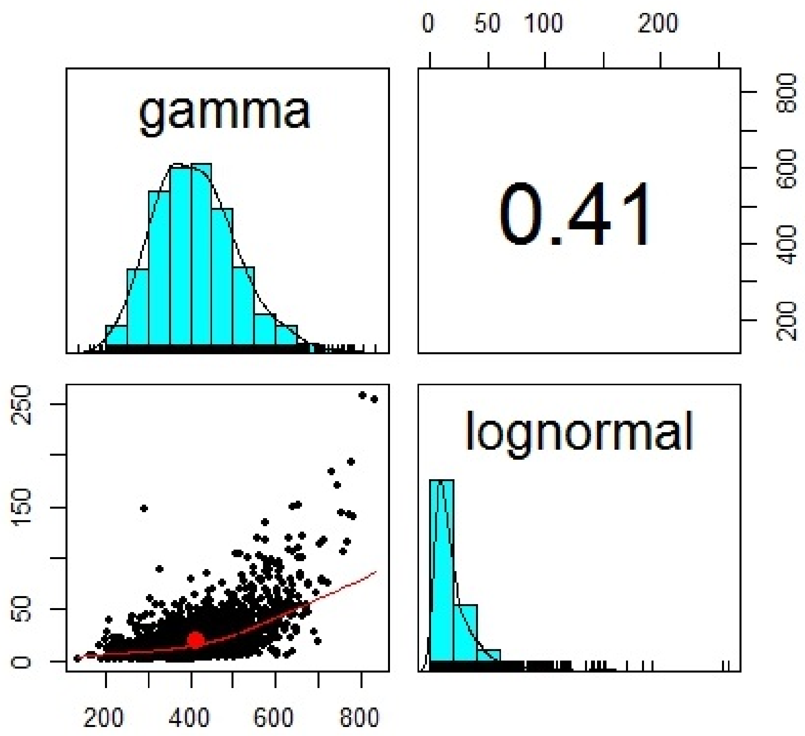

Figure 4: Panel Plot Tawn Copula with Gamma and Lognormal margins

$$C(x, y) = x^{1-a}y^{1-b}\exp\{-[(1 - a\ln x)^{0.98} + (-b\ln y)^{0.98}]^{1.02041}\}$$

These results indicate that Kendall's tau of the Frank copula for $\alpha = 18.55$(0.80) with gamma and lognormal marginals is very close to the empirical estimate(0.83). In addition, the contour and joint density plots suggest a strong lower tail dependence between MMR and TFR.

4 Conclusion

The empirical results revealed a strong correlation between indicators; MMR and TFR showed a positive association(0.83) whilst an inverse relation exist between MMR and GDP(-0.67) and TFR and GDP(-0.76). The contour and joint density plots from appendix A and B indicate a strong lower tail dependence for the bivariate Frank copula with Gamma and Lognormal margins whereas the Gumbel copula with Gamma and Lognormal margins shows strong upper tail dependence. These correlation figures tend to suggests that improved GDP as a consequence of improvement in socio-economic conditions of a Ghanaian mother tend to reduce Maternal Mortalities whilst increased fertility rates (population growth) turn to increase MMR. Generally, evidence has been drawn to improvement in GDP: logistics, number of skilled birth attendants, access to health fertilities and services, and general well-being as areas that require attention. Other indirect intervention schemes and programs such as family planing and contraceptive use targeted at reducing the high TFR also requires some considerable level of investment as well as stakeholders recommendation.

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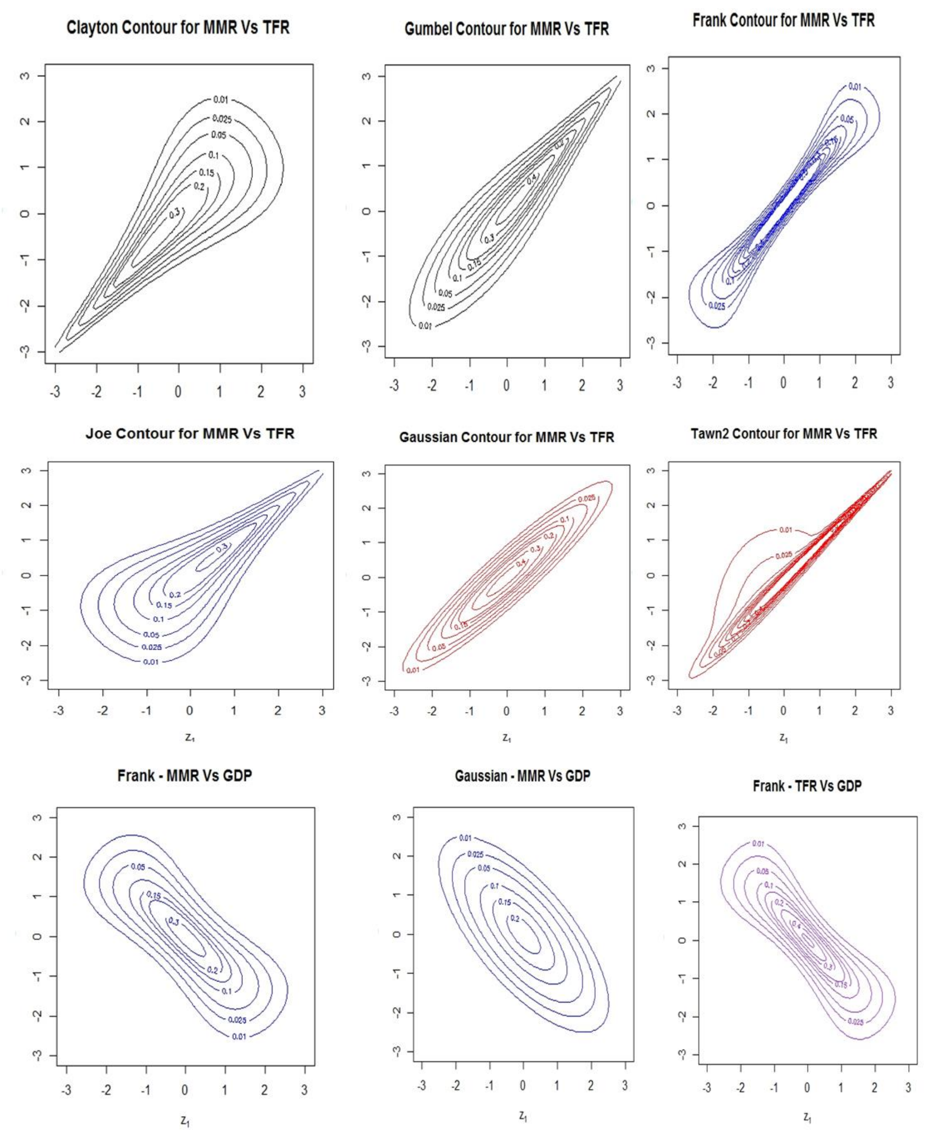
Appendix A: Contour plots



Appendix B: Density plots