(Kink; Kink; kink; Kink) and (Pulse; pulse; Pulse; pulse) Solutions of a Set of four Equations Modeled in a Nonlinear Hybrid Electrical Line with crosslink capacitor

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Abstract: The physics system that helps us in the study of this paper is a nonlinear hybrid electrical line with crosslink capacitor. Meaning it is composed of two different nonlinear hybrid parts Linked by capacitors with identical constant capacitance. We apply Kirchhoff laws to the circuit of the line to obtain new set of four nonlinear partial differential equations which describe the simultaneous dynamics of four solitary waves. Furthermore, we apply efficient mathematical methods based on the identification of coefficients of basic hyperbolic functions to construct exact solutions of those set of four nonlinear partial differential equations. The obtained results have enabled us to discover that, one of the two nonlinear hybrid electrical line with crosslink capacitor that we have modeled accepts the simultaneous propagation of a set of four solitary waves of type (Pulse; Pulse; Pulse; Pulse), while the other accepts the simultaneous propagation of a set of four solitary waves of type (Kink; Kink; Kink; Kink) when certain conditions we have established are respected. We ameliorate the quality of the signals by changing the sinusoidal waves that are supposed to propagate in the hybrid electrical lines with crosslink capacitor to solitary waves which are propagating in the new nonlinear hybrid electrical lines; we therefore, facilitate the choice of the type of line relative to the type of signal that we want to transmit.

Keywords: Hybrid electrical line, crosslink capacitor, construction, solitons solution, solitary wave, Nonlinear Partial Differential Equation, Kink, Pulse

1. Introduction

The signal propagated in the electrical lines where the parameters of its components are constant is a sinusoidal wave whose amplitude decreases exponentially and loses a lot of energy contrary to solitary wave signal which conserves its velocity, its shape and does not loses energy during its movement. Work has been carried out to study the hybrid line using numerical simulation with the goal of better matching to a resistive load. They projected that a hybrid line made of parallel plate with nonlinear capacitors and inductors could be developed to produce solitons with frequency between 1-2GHz [1, 2]. If solitons could be propagated in electrical lines, they will resist better on dissipation factors; for this reason, we have decided to carry out research on what means we could modify the component parameters of a hybrid electrical line with crosslink capacitor so that it accepts the propagation of solitary waves. We therefore define analytically the nonlinear flux linkage of inductors and the nonlinear charge of capacitors constituting the two parts linked by

capacitors in the line. The use of these definitions and the application of Kirchhoff laws to the circuit of nonlinear hybrid electrical line with crosslink capacitor has enabled us to model a set of four nonlinear partial differential equations which describe the dynamics of solitary waves in the line. To construct exact solitary wave solution of each set of four nonlinear partial differential equations, we have used the mathematical methods presented in [3-16] and particularly the Bogning-Djeumen Tchaho-Kofane method [17-22]. For one of the set of four nonlinear partial differential equations, we have obtained a solution which is a set of four solitary waves of type (Pulse; Pulse; Pulse; Pulse) and for the other we have obtained a solution which is a set of four solitary waves of type (Kink; Kink; Kink; Kink). Our work is developed in the following order: in section two, we model a nonlinear hybrid electrical line with crosslink capacitor; in section three we find the solitary wave solution of type (Kink; Kink; Kink; Kink); in section four we find the solitary wave solution of type (Pulse; Pulse; Pulse; Pulse). We conclude our work in section 5.

2. General modeling of nonlinear hybrid electrical line with crosslink capacitor.

Let us consider a nonlinear hybrid electrical line shown in figure 1. The line is constituted by a good number of identical networks numbered by the positive integer n. The network number n is constituted by a capacitor with capacitance C_0 which link the two nonlinear hybrid parts; two capacitors in which each of the charge q_1^n and q_2^n changes respectively in nonlinear manner in terms of the voltage u_1^n and u_2^n across each capacitor; two inductors in which each of the magnetic flux ϕ_1^n and ϕ_2^n changes respectively in nonlinear manner in terms of the current i_1^n and i_2^n that flow through each inductor.

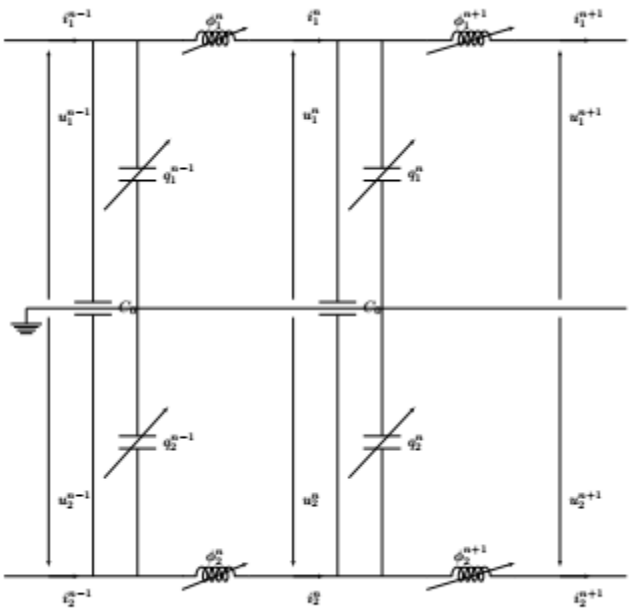

Figure 1: presentation of a nonlinear hybrid electrical line with crosslink capacitor.

Applying Kirchhoff's laws to the circuit shown in figure 1, we obtain the following equations:

$$u_1^n - u_1^{n-1} = -\frac{\partial \phi_1^n}{\partial t} \tag{1}$$

$$u_2^n - u_2^{n-1} = -\frac{\partial \phi_2^n}{\partial t} \tag{2}$$

$$i_1^n - i_1^{n+1} = C_0 \frac{\partial \left(u_1^n - u_2^n\right)}{\partial t} + \frac{\partial q_1^n}{\partial t} \tag{3}$$

$$i_2^n - i_2^{n+1} = -C_0 \frac{\partial \left(u_1^n - u_2^n\right)}{\partial t} + \frac{\partial q_2^n}{\partial t} \tag{4}$$

To obtain the continuum model, the left hand side of each equation (1); (2) ; (3) et (4) has to be approximated to a spatial partial derivative with respect to $x = nh$ which represents the distance measured from the beginning of the line. h represent the distance that separates two consecutive nodes and which is equivalent to the spatial sampling derivatives period. Using respectively Taylor expansion of u_1^{n-1} ; u_2^{n-1} ; i_1^{n+1} and i_2^{n+1} closely to u_1^n; u_2^n ; i_1^n and i_2^n by considering the terms till fourth order we obtain the set of four partial differential equations in the following manner:

$$\begin{cases} \dfrac{h^4}{24}\dfrac{\partial^4 u_1^n}{\partial x^4} - \dfrac{h^3}{6}\dfrac{\partial^3 u_1^n}{\partial x^3} + \dfrac{h^2}{2}\dfrac{\partial^2 u_1^n}{\partial x^2} - h\dfrac{\partial u_1^n}{\partial x} - \dfrac{\partial \phi_1^n}{\partial t} = 0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 u_2^n}{\partial x^4} - \dfrac{h^3}{6}\dfrac{\partial^3 u_2^n}{\partial x^3} + \dfrac{h^2}{2}\dfrac{\partial^2 u_2^n}{\partial x^2} - h\dfrac{\partial u_2^n}{\partial x} - \dfrac{\partial \phi_2^n}{\partial t} = 0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 i_1^n}{\partial x^4} + \dfrac{h^3}{6}\dfrac{\partial^3 i_1^n}{\partial x^3} + \dfrac{h^2}{2}\dfrac{\partial^2 i_1^n}{\partial x^2} + h\dfrac{\partial i_1^n}{\partial x} + C_0\dfrac{\partial \left(u_1^n - u_2^n\right)}{\partial t} + \dfrac{\partial q_1^n}{\partial t} = 0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 i_2^n}{\partial x^4} + \dfrac{h^3}{6}\dfrac{\partial^3 i_2^n}{\partial x^3} + \dfrac{h^2}{2}\dfrac{\partial^2 i_2^n}{\partial x^2} + h\dfrac{\partial i_2^n}{\partial x} - C_0\dfrac{\partial \left(u_1^n - u_2^n\right)}{\partial t} + \dfrac{\partial q_2^n}{\partial t} = 0 \end{cases} \tag{5}$$

Finally, we obtain the continuum model of the nonlinear hybrid electrical line with crosslink capacitor presented in figure1 by the set of four nonlinear partial differential equations below:

$$\begin{cases} \dfrac{h^4}{24}\dfrac{\partial^4 u_1(x,t)}{\partial x^4}-\dfrac{h^3}{6}\dfrac{\partial^3 u_1(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 u_1(x,t)}{\partial x^2} \\ -h\dfrac{\partial u_1(x,t)}{\partial x}-\dfrac{\partial \phi_1\left(i_1(x,t)\right)}{\partial t}=0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 u_2(x,t)}{\partial x^4}-\dfrac{h^3}{6}\dfrac{\partial^3 u_2(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 u_2(x,t)}{\partial x^2} \\ -h\dfrac{\partial u_2(x,t)}{\partial x}-\dfrac{\partial \phi_2\left(i_2(x,t)\right)}{\partial t}=0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 i_1(x,t)}{\partial x^4}+\dfrac{h^3}{6}\dfrac{\partial^3 i_1(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 i_1(x,t)}{\partial x^2}+h\dfrac{\partial i_1(x,t)}{\partial x} \\ +C_0\dfrac{\partial\left(u_1(x,t)-u_2(x,t)\right)}{\partial t}+\dfrac{\partial q_1\left(u_1(x,t)\right)}{\partial t}=0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 i_2(x,t)}{\partial x^4}+\dfrac{h^3}{6}\dfrac{\partial^3 i_2(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 i_2(x,t)}{\partial x^2}+h\dfrac{\partial i_2(x,t)}{\partial x} \\ -C_0\dfrac{\partial\left(u_1(x,t)-u_2(x,t)\right)}{\partial t}+\dfrac{\partial q_2\left(u_2(x,t)\right)}{\partial t}=0 \end{cases} \tag{6}$$

3. Construction of a set of four solitary wave solutions of type (Kink ; Kink; Kink ; Kink) relative to general differential equation (6)

We define each of nonlinear charges $q_1\left(u_1(x,t)\right)$, $q_2\left(u_2(x,t)\right)$ of the capacitors and each of nonlinear magnetic flux linkage $\phi_1\left(i_1(x,t)\right)$, $\phi_2\left(i_2(x,t)\right)$ of the inductors under the analytical shape given below:

$$\begin{cases} \phi_1\left(i_1(x,t)\right)=E_1 i_1(x,t)+E_2 i_1^2(x,t)+E_3 i_1^3(x,t)+E_4 i_1^4(x,t) \\ \phi_2\left(i_2(x,t)\right)=F_1 i_2(x,t)+F_2 i_2^2(x,t)+F_3 i_2^3(x,t)+F_4 i_2^4(x,t) \\ q_1\left(u_1(x,t)\right)=A_1 u_1(x,t)+A_2 u_1^2(x,t)+A_3 u_1^3(x,t)+A_4 u_1^4(x,t) \\ q_2\left(u_2(x,t)\right)=B_1 u_2(x,t)+B_2 u_2^2(x,t)+B_3 u_2^3(x,t)+B_4 u_2^4(x,t) \end{cases} \tag{7}$$

With E_1 ; E_2 ; E_3 ; E_4 ; F_1 ; F_2 ; F_3 ; F_4 ; A_1 ; A_2 ; A_3 ; A_4 ; B_1 ; B_2 ; B_3 and B_4 are non-nil real numbers which will be chosen conveniently. Let us note that E_1 and F_1 stand for inductance, E_2 and F_2 stand for inductance per unit current, E_3 and F_3 stand for inductance per unit current of power two, E_4 and F_4 stand for inductance per unit current of power three, A_1 and B_1 stand for capacitance, A_2 and B_2 stand for capacitance per unit voltage, A_3 and B_3 stand for capacitance per unit voltage of power two, A_4 and B_4 stand for capacitance per unit voltage of power three.

By substituting each of the nonlinear charge $q_1\left(u_1(x,t)\right)$, $q_2\left(u_2(x,t)\right)$ and each of the nonlinear magnetic flux $\phi_1\left(i_1(x,t)\right)$, $\phi_2\left(i_2(x,t)\right)$ of (7) in (6) we obtain the set of four nonlinear partial differential equation written as:

$$\begin{cases} \dfrac{h^4}{24}\dfrac{\partial^4 u_1(x,t)}{\partial x^4}-\dfrac{h^3}{6}\dfrac{\partial^3 u_1(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 u_1(x,t)}{\partial x^2}-h\dfrac{\partial u_1(x,t)}{\partial x} \\ +\left(-E_1-2E_2 i_1(x,t)-3E_3 i_1^2(x,t)-4E_4 i_1^3(x,t)\right)\dfrac{\partial i_1(x,t)}{\partial t}=0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 u_2(x,t)}{\partial x^4}-\dfrac{h^3}{6}\dfrac{\partial^3 u_2(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 u_2(x,t)}{\partial x^2}-h\dfrac{\partial u_2(x,t)}{\partial x} \\ +\left(-F_1-2F_2 i_2(x,t)-3F_3 i_2^2(x,t)-4F_4 i_2^3(x,t)\right)\dfrac{\partial i_2(x,t)}{\partial t}=0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 i_1(x,t)}{\partial x^4}+\dfrac{h^3}{6}\dfrac{\partial^3 i_1(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 i_1(x,t)}{\partial x^2}+h\dfrac{\partial i_1(x,t)}{\partial x}-C_0\dfrac{\partial u_2(x,t)}{\partial t} \\ +\left(C_0+A_1+2A_2 u_1(x,t)+3A_3 u_1^2(x,t)+4A_4 u_1^3(x,t)\right)\dfrac{\partial u_1(x,t)}{\partial t}=0 \\ \dfrac{h^4}{24}\dfrac{\partial^4 i_2(x,t)}{\partial x^4}+\dfrac{h^3}{6}\dfrac{\partial^3 i_2(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 i_2(x,t)}{\partial x^2}+h\dfrac{\partial i_2(x,t)}{\partial x}-C_0\dfrac{\partial u_1(x,t)}{\partial t} \\ +\left(C_0+B_1+2B_2 u_2(x,t)+3B_3 u_2^2(x,t)+4B_4 u_2^3(x,t)\right)\dfrac{\partial u_2(x,t)}{\partial t}=0 \end{cases} \quad (8)$$

Let us use Bogning-Djeumen Tchaho-Kofane method [17-22] to come out with the solution of (8) under the analytical shape below:

$$\begin{pmatrix} u_1(x,t)=a\tanh(kx-vt) \\ u_2(x,t)=b\tanh(kx-vt) \\ i_1(x,t)=e\tanh(kx-vt) \\ i_2(x,t)=f\tanh(kx-vt) \end{pmatrix} \quad (9)$$

Where a, b, e, f, are wave amplitudes; k the wave vector and v the velocity which are non-zero real numbers to be determined in terms of modeled line parameters. Replacing $u_1(x,t)$; $u_1(x,t)$; $i_1(x,t)$ et $i_2(x,t)$ given by (9) in (8) we yield the following set of four equations which are written in a simplified form:

$$
\begin{cases}
\left(3E_3e^3v - hak - \frac{2}{3}h^3ak^3 + E_1ev\right)\frac{1}{\cosh^2(kx-vt)} \\
+\left(2E_2e^2v - \frac{1}{3}h^4ak^4 - h^2ak^2 + 4E_4e^4v\right)\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)} \\
+\left(h^4ak^4 - 4E_4e^4v\right)\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)} + \left(-3E_3e^3v + h^3ak^3\right)\frac{1}{\cosh^4(kx-vt)} = 0 \\
\left(3F_3f^3v - hbk - \frac{2}{3}h^3bk^3 + F_1fv\right)\frac{1}{\cosh^2(kx-vt)} \\
+\left(2F_2f^2v - \frac{1}{3}h^4bk^4 - h^2bk^2 + 4F_4f^4v\right)\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)} \\
+\left(h^4bk^4 - 4F_4f^4v\right)\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)} + \left(-3F_3f^3v + h^3bk^3\right)\frac{1}{\cosh^4(kx-vt)} = 0 \\
\left(-3A_3a^3v + hek + \frac{2}{3}h^3ek^3 - A_1av - C_0av + C_0bv\right)\frac{1}{\cosh^2(kx-vt)} \\
+\left(3A_3a^3v - h^3ek^3\right)\frac{1}{\cosh^4(kx-vt)} + \left(h^4ek^4 + 4A_4a^4v\right)\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)} \\
+\left(-2A_2a^2v - \frac{1}{3}h^4ek^4 - h^2ek^2 - 4A_4a^4v\right)\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)} = 0 \\
\left(-3B_3b^3v + hfk + \frac{2}{3}h^3fk^3 - B_1bv - C_0bv + C_0av\right)\frac{1}{\cosh^2(kx-vt)} \\
+\left(3B_3b^3v - h^3fk^3\right)\frac{1}{\cosh^4(kx-vt)}\left(h^4fk^4 + 4B_4b^4v\right)\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)} \\
+\left(-2B_2b^2v - \frac{1}{3}h^4fk^4 - h^2fk^2 - 4B_4b^4v\right)\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)} + = 0
\end{cases}
\tag{10}
$$

The set of equations (10) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permits us to obtain the following set of sixteen equations:

$$
\begin{cases}
3E_3e^3v - hak - \dfrac{2}{3}h^3ak^3 + E_1ev = 0 \\
2E_2e^2v - \dfrac{1}{3}h^4ak^4 - h^2ak^2 + 4E_4e^4v = 0 \\
h^4ak^4 - 4E_4e^4v = 0 \\
-3E_3e^3v + h^3ak^3 = 0 \\
3F_3f^3v - hbk - \dfrac{2}{3}h^3bk^3 + F_1fv = 0 \\
2F_2f^2v - \dfrac{1}{3}h^4bk^4 - h^2bk^2 + 4F_4f^4v = 0 \\
h^4bk^4 - 4F_4f^4v = 0 \\
-3F_3f^3v + h^3bk^3 = 0 \\
-3A_3a^3v + hek + \dfrac{2}{3}h^3ek^3 - A_1av - C_0av + C_0bv = 0 \\
-2A_2a^2v - \dfrac{1}{3}h^4ek^4 - h^2ek^2 - 4A_4a^4v = 0 \\
3A_3a^3v - h^3ek^3 = 0 \\
h^4ek^4 + 4A_4a^4v = 0 \\
-3B_3b^3v + hfk + \dfrac{2}{3}h^3fk^3 - B_1bv - C_0bv + C_0av = 0 \\
-2B_2b^2v - \dfrac{1}{3}h^4fk^4 - h^2fk^2 - 4B_4b^4v = 0 \\
3B_3b^3v - h^3fk^3 = 0 \\
h^4fk^4 + 4B_4b^4v = 0
\end{cases}
\tag{11}
$$

Haven solved the set of equation (11), it has permitted us to present in (12) the solution with conditions of the set of four nonlinear partial differential equations obtained in (8) which model the dynamic of a set of four solitary wave of type (Kink; Kink; Kink; Kink):

$$
a = \frac{\sqrt{-48A_2A_4 + 54A_3^2}}{8A_4} \; ; \; e = \frac{\left(A_3E_3^2\right)^{\frac{1}{4}}\sqrt{-48A_2A_4 + 54A_3^2}}{8E_3A_4} \; ; \; f = \frac{B_3^4A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4 + 54A_3^2}}{8E_3A_3^4B_4^3} \; ;
$$

$$
b = \frac{E_3}{B_3\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{B_3^6\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4 + 54A_3^2\right)^{\frac{3}{2}}}{512A_3^3E_3^3B_4^3}\right)^{\frac{1}{3}} \; ; \; v = \frac{-8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4 + 54A_3^2}}{81E_3A_3^4} \; ;
$$

$$k=\frac{1}{\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216h^3A_3^3}\right)^{\frac{1}{3}}\ ;\ E_4=\frac{h^4ak^4}{4e^4v}\ ;\ F_3=\frac{h^3bk^3}{4f^3v}\ ;\ F_4=\frac{h^4bk^4}{4f^4v}\ ;$$

$$A_1=\frac{-h^3ek^3a^3+3heka^3-3C_0va^4+3C_0va^3b}{3va^4}\ ;\ B_1=\frac{-h^3fk^3b^3+3hfkb^3-3C_0vb^4+3C_0vb^3a}{3vb^4}\ ;$$

$$E_1=\frac{-hak\left(-3e^3+h^2k^2e^3\right)}{3e^4v}\ ;\qquad E_2=\frac{-hak\left(-\frac{3}{2}hke^2+h^3k^3e^2\right)}{3e^4v}\ ;\qquad F_1=\frac{-hbk\left(-3f^3+h^2k^2f^3\right)}{3f^4v}\ ;$$

$$F_2=\frac{-hbk\left(-\frac{3}{2}hkf^2+h^3k^3f^2\right)}{3f^4v}\ ;\ A_3<0\ ;\ E_3<0\ ;\ 54A_3^2>48A_2A_4\ ;$$

$$\left(\begin{array}{l}
u_1(x,t)=\frac{\sqrt{-48A_2A_4+54A_3^2}}{8A_4}\tanh\left(\begin{array}{l}\frac{1}{\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216h^3A_3^3}\right)^{\frac{1}{3}}x\\ +\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)\\
u_2(x,t)=\frac{E_3}{B_3\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{B_3^6\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{512A_3^3E_3^3B_4^3}\right)^{\frac{1}{3}}\tanh\left(\begin{array}{l}\frac{1}{\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216h^3A_3^3}\right)^{\frac{1}{3}}x\\ +\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)\\
i_1(x,t)=\frac{\left(A_3E_3^2\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{8E_3A_4}\tanh\left(\begin{array}{l}\frac{1}{\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216h^3A_3^3}\right)^{\frac{1}{3}}x\\ +\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)\\
i_2(x,t)=\frac{B_3^4A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{8E_3A_3^4B_4^3}\tanh\left(\begin{array}{l}\frac{1}{\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216h^3A_3^3}\right)^{\frac{1}{3}}x\\ +\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)
\end{array}\right)\qquad(12)$$

For the values of the following parameters : $A_2 = 37\times10^{-12}\,F/V$, $A_3 = -7,28\times10^{-10}\,F/V^2$, $A_4 = -30\times10^{-12}\,F/V^3$, $E_3 = -47\times10^{-6}\,H/A^2$, $B_3 = 7,28\times10^{-15}\,F/V^2$, $B_4 = 47\times10^{-19}\,F/V^3$, $h = -10^{-4}\,m$, the expressions of four Kink solitons (12) can be re-written as $u_1 = 22,31\tanh\left(12258,84x + 1,06\times10^5 t\right)$, $u_2 = 1424,11\tanh\left(12258,84x + 1,06\times10^5 t\right)$, $i_1 = -1,39\tanh\left(12258,84x + 1,06\times10^5 t\right)$, $i_2 = 3,63\tanh\left(12258,84x + 1,06\times10^5 t\right)$. This permits to obtain in figure 2 the representation of real profile of those four Kink solitons.

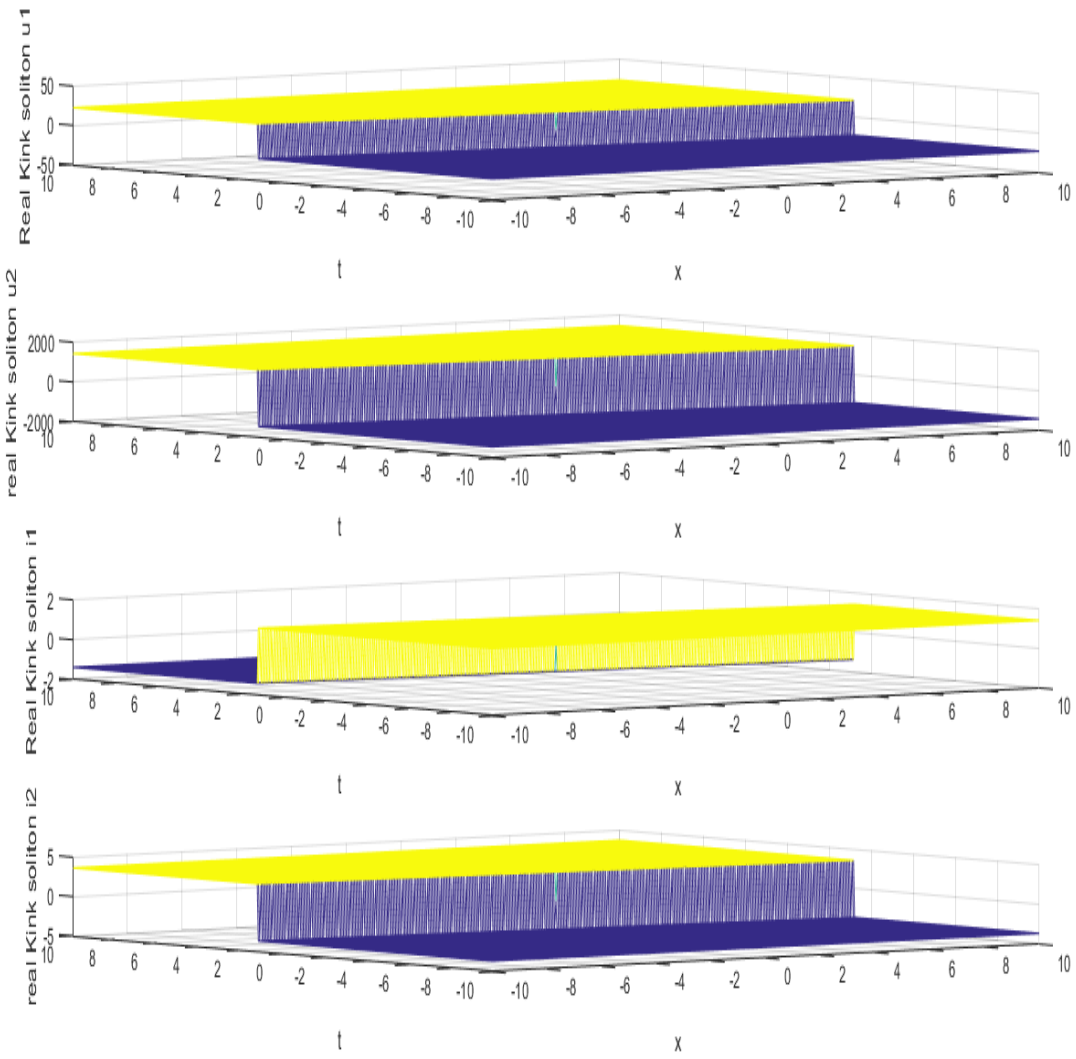

Figure 2: Real profile of the four Kink solitons

This representation shows real profile of the four Kink solitons which are topological solitons since the properties of their media are not the same at infinity.

4. Construction of a set of four solitary wave solutions of type (Pulse; Pulse; Pulse; Pulse) relative to general differential equation (6)

We define each of nonlinear charges $q_1\left(u_1(x,t)\right)$, $q_2\left(u_2(x,t)\right)$ of the capacitors and each of nonlinear magnetic flux linkage $\phi_1\left(i_1(x,t)\right)$, $\phi_2\left(i_2(x,t)\right)$ of the inductors under the analytical shape given below:

$$\begin{cases} \phi_1\left(i_1(x,t)\right) = E_1 i_1(x,t) + E_2 i_1^3(x,t) + \left(E_3 i_1(x,t) + E_4 i_1^3(x,t)\right)\sqrt{1-\dfrac{i_1^2(x,t)}{E_0^2}} \\ \phi_2\left(i_2(x,t)\right) = F_1 i_2(x,t) + F_2 i_2^3(x,t) + \left(F_3 i_2(x,t) + F_4 i_2^3(x,t)\right)\sqrt{1-\dfrac{i_2^2(x,t)}{F_0^2}} \\ q_1\left(u_1(x,t)\right) = A_1 u_1(x,t) + A_2 u_1^3(x,t) + \left(A_3 u_1(x,t) + A_4 u_1^3(x,t)\right)\sqrt{1-\dfrac{u_1^2(x,t)}{A_0^2}} \\ q_2\left(u_2(x,t)\right) = B_1 u_2(x,t) + B_2 u_2^3(x,t) + \left(B_3 u_2(x,t) + B_4 u_2^3(x,t)\right)\sqrt{1-\dfrac{u_2^2(x,t)}{B_0^2}} \end{cases} \quad (13)$$

With $|E_0| > |i_1(x,t)|$; $|F_0| > |i_2(x,t)|$; $|A_0| > |u_1(x,t)|$; $|B_0| > |u_2(x,t)|$. E_1 ; E_2 ; E_3 ; E_4 ; F_1 ; F_2 ; F_3 ; F_4 ; A_1 ; A_2 ; A_3 ; A_4 ; B_1 ; B_2 ; B_3 and B_4 are non-nil real numbers which will be chosen conveniently. Let us note that E_1, F_1, E_3, F_3 stand for inductance ; E_2, F_2, E_4, F_4 stand for inductance per unit current of power two; A_1, B_1, A_3, B_3 stand for capacitance ; A_2, B_2, A_4 and B_4 stand for capacitance per unit voltage of power two. By substituting each of the nonlinear charge $q_1\left(u_1(x,t)\right)$, $q_2\left(u_2(x,t)\right)$ and each of the nonlinear magnetic flux $\phi_1\left(i_1(x,t)\right)$, $\phi_2\left(i_2(x,t)\right)$ of (13) in (6) we obtain the set of four nonlinear partial differential equation written as:

$$
\begin{cases}
\dfrac{h^4}{24}\dfrac{\partial^4 u_1(x,t)}{\partial x^4}-\dfrac{h^3}{6}\dfrac{\partial^3 u_1(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 u_1(x,t)}{\partial x^2}-h\dfrac{\partial u_1(x,t)}{\partial x} \\
+\left(-E_1-3E_2 i_1^2(x,t)-\left(E_3+3E_4 i_1^2(x,t)\right)\sqrt{1-\dfrac{i_1^2(x,t)}{E_0^2}}+\dfrac{E_3 i_1^2(x,t)+E_4 i_1^4(x,t)}{E_0^2\sqrt{1-\dfrac{i_1^2(x,t)}{E_0^2}}}\right)\dfrac{\partial i_1(x,t)}{\partial t}=0 \\
\dfrac{h^4}{24}\dfrac{\partial^4 u_2(x,t)}{\partial x^4}-\dfrac{h^3}{6}\dfrac{\partial^3 u_2(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 u_2(x,t)}{\partial x^2}-h\dfrac{\partial u_2(x,t)}{\partial x} \\
+\left(-F_1-3F_2 i_2^2(x,t)-\left(F_3+3F_4 i_2^2(x,t)\right)\sqrt{1-\dfrac{i_2^2(x,t)}{F_0^2}}+\dfrac{F_3 i_2^2(x,t)+F_4 i_2^4(x,t)}{F_0^2\sqrt{1-\dfrac{i_2^2(x,t)}{F_0^2}}}\right)\dfrac{\partial i_2(x,t)}{\partial t}=0 \\
\dfrac{h^4}{24}\dfrac{\partial^4 i_1(x,t)}{\partial x^4}+\dfrac{h^3}{6}\dfrac{\partial^3 i_1(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 i_1(x,t)}{\partial x^2}+h\dfrac{\partial i_1(x,t)}{\partial x}-C_0\dfrac{\partial u_2(x,t)}{\partial t} \\
+\left(C_0+A_1+2A_2 u_1^2(x,t)+\left(A_3+3A_4 u_1^2(x,t)\right)\sqrt{1-\dfrac{u_1^2(x,t)}{A_0^2}}-\dfrac{A_3 u_1^2(x,t)+A_4 u_1^4(x,t)}{A_0^2\sqrt{1-\dfrac{u_1^2(x,t)}{A_0^2}}}\right)\dfrac{\partial u_1(x,t)}{\partial t}=0 \\
\dfrac{h^4}{24}\dfrac{\partial^4 i_2(x,t)}{\partial x^4}+\dfrac{h^3}{6}\dfrac{\partial^3 i_2(x,t)}{\partial x^3}+\dfrac{h^2}{2}\dfrac{\partial^2 i_2(x,t)}{\partial x^2}+h\dfrac{\partial i_2(x,t)}{\partial x}-C_0\dfrac{\partial u_1(x,t)}{\partial t} \\
+\left(C_0+B_1+2B_2 u_2^2(x,t)+\left(B_3+3B_4 u_2^2(x,t)\right)\sqrt{1-\dfrac{u_2^2(x,t)}{B_0^2}}-\dfrac{B_3 u_2^2(x,t)+B_4 u_2^4(x,t)}{B_0^2\sqrt{1-\dfrac{u_2^2(x,t)}{B_0^2}}}\right)\dfrac{\partial u_2(x,t)}{\partial t}=0
\end{cases}
\tag{14}
$$

Let us use Bogning-Djeumen Tchaho-Kofane method [17-22] to come out with the solution of (14) under the analytical shape below:

$$
\begin{pmatrix}
u_1(x,t)=a\,\mathrm{sech}(kx-vt) \\
u_2(x,t)=b\,\mathrm{sech}(kx-vt) \\
i_1(x,t)=e\,\mathrm{sech}(kx-vt) \\
i_2(x,t)=f\,\mathrm{sech}(kx-vt)
\end{pmatrix}
\tag{15}
$$

Where a, b, e, f, are wave amplitudes; k the wave vector and v the velocity which non-zero real numbers to be determined in terms of modeled line parameters. Replacing $u_1(x,t)$; $u_1(x,t)$;

$i_1(x,t)$ et $i_2(x,t)$ given by (15) in (14) we yield the following set of four equations which are written in a simplified form $a = A_0$; $b = B_0$; $e = E_0$ et $f = F_0$:

$$
\begin{cases}
\left(-20h^4A_0k^4 + 48E_0vE_3 - 72E_0^3vE_4 - 24h^2A_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\
+\left(h^4A_0k^4 + 12h^2A_0k^2 - 24E_0vE_3\right)\sinh(kx-vt)\cosh^4(kx-vt) + \left(96E_0^3vE_4 + 24h^4A_0k^4\right)\sinh(kx-vt) \\
+\left(24h^3A_0k^3 + 72E_0^3vE_2\right)\cosh(kx-vt) + \left(-72E_0^3vE_2 - 24hA_0k + 24E_0vE_1 - 28h^3A_0k^3\right)\cosh^3(kx-vt) \\
+\left(24hA_0k - 24E_0vE_1 + 4h^3A_0k^3\right)\cosh^5(kx-vt) = 0 \\
\left(-20h^4B_0k^4 + 48F_0vF_3 - 72F_0^3vF_4 - 24h^2B_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\
+\left(h^4B_0k^4 + 12h^2B_0k^2 - 24F_0vF_3\right)\sinh(kx-vt)\cosh^4(kx-vt) + \left(96F_0^3vF_4 + 24h^4B_0k^4\right)\sinh(kx-vt) \\
+\left(24h^3B_0k^3 + 72F_0^3vF_2\right)\cosh(kx-vt) + \left(-72F_0^3vF_2 - 24hB_0k + 24F_0vF_1 - 28h^3B_0k^3\right)\cosh^3(kx-vt) \\
+\left(24hB_0k - 24F_0vF_1 + 4h^3B_0k^3\right)\cosh^5(kx-vt) = 0 \\
\left(-20h^4E_0k^4 - 48A_0vA_3 + 72A_0^3vA_4 - 24h^2E_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\
+\left(-24h^3E_0k^3 - 72A_0^3vA_2\right)\cosh(kx-vt) + \left(-96A_0^3vA_4 + 24h^4E_0k^4\right)\sinh(kx-vt) \\
+\left(h^4E_0k^4 + 12h^2E_0k^2 + 24A_0vA_3\right)\sinh(kx-vt)\cosh^4(kx-vt) \\
+\left(72A_0^3vA_2 + 24hE_0k - 24A_0vA_1 + 28h^3E_0k^3 - 24A_0vC_0 + 24B_0vC_0\right)\cosh^3(kx-vt) \\
+\left(-24hE_0k + 24A_0vA_1 - 4h^3E_0k^3 + 24A_0vC_0 - 24B_0vC_0\right)\cosh^5(kx-vt) = 0 \\
\left(-20h^4F_0k^4 - 48B_0vB_3 + 72B_0^3vB_4 - 24h^2F_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\
+\left(-24h^3F_0k^3 - 72B_0^3vB_2\right)\cosh(kx-vt) + \left(-96B_0^3vB_4 + 24h^4F_0k^4\right)\sinh(kx-vt) \\
+\left(h^4F_0k^4 + 12h^2F_0k^2 + 24B_0vB_3\right)\sinh(kx-vt)\cosh^4(kx-vt) \\
+\left(72B_0^3vB_2 + 24hF_0k - 24B_0vB_1 + 28h^3F_0k^3 - 24B_0vC_0 + 24A_0vC_0\right)\cosh^3(kx-vt) \\
+\left(-24hF_0k + 24B_0vB_1 - 4h^3F_0k^3 + 24B_0vC_0 - 24A_0vC_0\right)\cosh^5(kx-vt) = 0
\end{cases}
\tag{16}
$$

The set of equations (16) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permits us to obtain the following set of twenty four equations:

$$
\begin{cases}
-20h^4A_0k^4+48E_0vE_3-72E_0^3vE_4-24h^2A_0k^2=0\\
h^4B_0k^4+12h^2B_0k^2-24F_0vF_3=0\\
96E_0^3vE_4+24h^4A_0k^4=0\\
24h^3A_0k^3+72E_0^3vE_2=0\\
-72E_0^3vE_2-24hA_0k+24E_0vE_1-28h^3A_0k^3=0\\
24hA_0k-24E_0vE_1+4h^3A_0k^3=0\\
-20h^4B_0k^4+48F_0vF_3-72F_0^3vF_4-24h^2B_0k^2=0\\
h^4B_0k^4+12h^2B_0k^2-24F_0vF_3=0\\
96F_0^3vF_4+24h^4B_0k^4=0\\
24h^3B_0k^3+72F_0^3vF_2=0\\
-72F_0^3vF_2-24hB_0k+24F_0vF_1-28h^3B_0k^3=0\\
24hB_0k-24F_0vF_1+4h^3B_0k^3=0\\
-20h^4E_0k^4-48A_0vA_3+72A_0^3vA_4-24h^2E_0k^2=0\\
-24h^3E_0k^3-72A_0^3vA_2=0\\
-96A_0^3vA_4+24h^4E_0k^4=0\\
h^4E_0k^4+12h^2E_0k^2+24A_0vA_3=0\\
72A_0^3vA_2+24hE_0k-24A_0vA_1+28h^3E_0k^3-24A_0vC_0+24B_0vC_0=0\\
-24hE_0k+24A_0vA_1-4h^3E_0k^3+24A_0vC_0-24B_0vC_0=0\\
-20h^4F_0k^4-48B_0vB_3+72B_0^3vB_4-24h^2F_0k^2=0\\
-24h^3F_0k^3-72B_0^3vB_2=0\\
-96B_0^3vB_4+24h^4F_0k^4=0\\
h^4F_0k^4+12h^2F_0k^2+24B_0vB_3=0\\
72B_0^3vB_2+24hF_0k-24B_0vB_1+28h^3F_0k^3-24B_0vC_0+24A_0vC_0=0\\
-24hF_0k+24B_0vB_1-4h^3F_0k^3+24B_0vC_0-24A_0vC_0=0
\end{cases}
\tag{17}
$$

Haven solved the set of equation (17), it has permitted us to present in (18) the solution with conditions of the set of four nonlinear partial differential equations obtained in (14) which model the dynamic of a set of four solitary wave of type (pulse ; pulse ; pulse ; pulse):

$$a=A_0\ ;\quad b=B_0\ ;\quad e=E_0\ ;\quad f=F_0\ ;\quad k=\frac{A_0}{E_0h}\left(\frac{-64A_4^3E_0^3}{27A_2^3A_0^3}\right)^{\frac{1}{3}}\ ;\quad v=\frac{64A_4^3E_0}{81A_2^4A_0^3}\ ;\quad A_2A_4<0\ ;$$

$$B_2=-\frac{h^3fk^3}{3B_0^2v}\ ;\ B_4=\frac{h^4fk^4}{4B_0^3v}\ ;\ E_2=-\frac{h^3A_0k^3}{3e^2v}\ ;\ E_4=-\frac{h^4A_0k^4}{4e^3v}\ ;\ F_2=-\frac{h^3B_0k^3}{3f^2v}\ ;\ F_4=-\frac{h^4B_0k^4}{4f^3v}\ ;$$

$$A_1=-C_0+\frac{C_0B_0}{A_0}+\frac{h^3ek^3}{6A_0v}+\frac{hek}{A_0v}\ ;\qquad A_3=-\frac{h^4ek^4}{24A_0v}-\frac{h^2ek^2}{2A_0v}\ ;\qquad B_1=-C_0+\frac{C_0A_0}{B_0}+\frac{h^3fk^3}{6B_0v}+\frac{hfk}{B_0v}\ ;$$

$$B_3=-\frac{h^4 fk^4}{24B_0 v}-\frac{h^2 fk^2}{2B_0 v}\ ;\quad E_1=\frac{hA_0 k}{ev}+\frac{h^3 A_0 k^3}{6ev}\ ;\quad E_3=\frac{h^2 A_0 k^2}{2ev}+\frac{h^4 A_0 k^4}{24ev}\ ;\quad F_1=\frac{hB_0 k}{fv}+\frac{h^3 B_0 k^3}{6fv}\ ;$$

$$F_3=\frac{h^2 B_0 k^2}{2fv}+\frac{h^4 B_0 k^4}{24fv}\ ;\left(\begin{array}{l} u_1(x,t)=A_0\,\mathrm{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3E_0^3}{27A_2^3A_0^3}\right)^{\frac{1}{3}}x-\frac{64A_4^3E_0}{81A_2^4A_0^3}t\right)\\ u_2(x,t)=B_0\,\mathrm{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3E_0^3}{27A_2^3A_0^3}\right)^{\frac{1}{3}}x-\frac{64A_4^3E_0}{81A_2^4A_0^3}t\right)\\ i_1(x,t)=E_0\,\mathrm{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3E_0^3}{27A_2^3A_0^3}\right)^{\frac{1}{3}}x-\frac{64A_4^3E_0}{81A_2^4A_0^3}t\right)\\ i_2(x,t)=F_0\,\mathrm{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3E_0^3}{27A_2^3A_0^3}\right)^{\frac{1}{3}}x-\frac{64A_4^3E_0}{81A_2^4A_0^3}t\right) \end{array}\right) \qquad (18)$$

In mathematical domain, the nonlinear hybrid electrical line with crosslink capacitor presented in figure 1 has permitted in the one hand to discover in (8) a set of four nonlinear partial differential equations which have for exact solution a set of four solitary waves given in (12) and on the other hand to discover in (14) another set of four nonlinear partial differential equations which have for exact solution another set of four solitary waves given in (18).

For the values of the following parameters : $A_2=37\times10^{-11}\,F/V^2$, $A_4=-30\times10^{-12}\,F/V^2$, $A_0=10V$, $B_0=-20V$, $E_0=0,3A$, $F_0=-5A$, $h=10^{-4}$, the expressions of four Pulse solitons (18) can be re-written as $u_1=10\,\mathrm{sech}(1081,08x+341,48t)$, $u_2=-20\,\mathrm{sech}(1081,08x+341,48t)$, $i_1=0,3\,\mathrm{sech}(1081,08x+341,48t)$, $i_2=-5\,\mathrm{sech}(1081,08x+341,48t)$. This permits to obtain in figure 3 the representation of real profile of those four Pulse solitons.

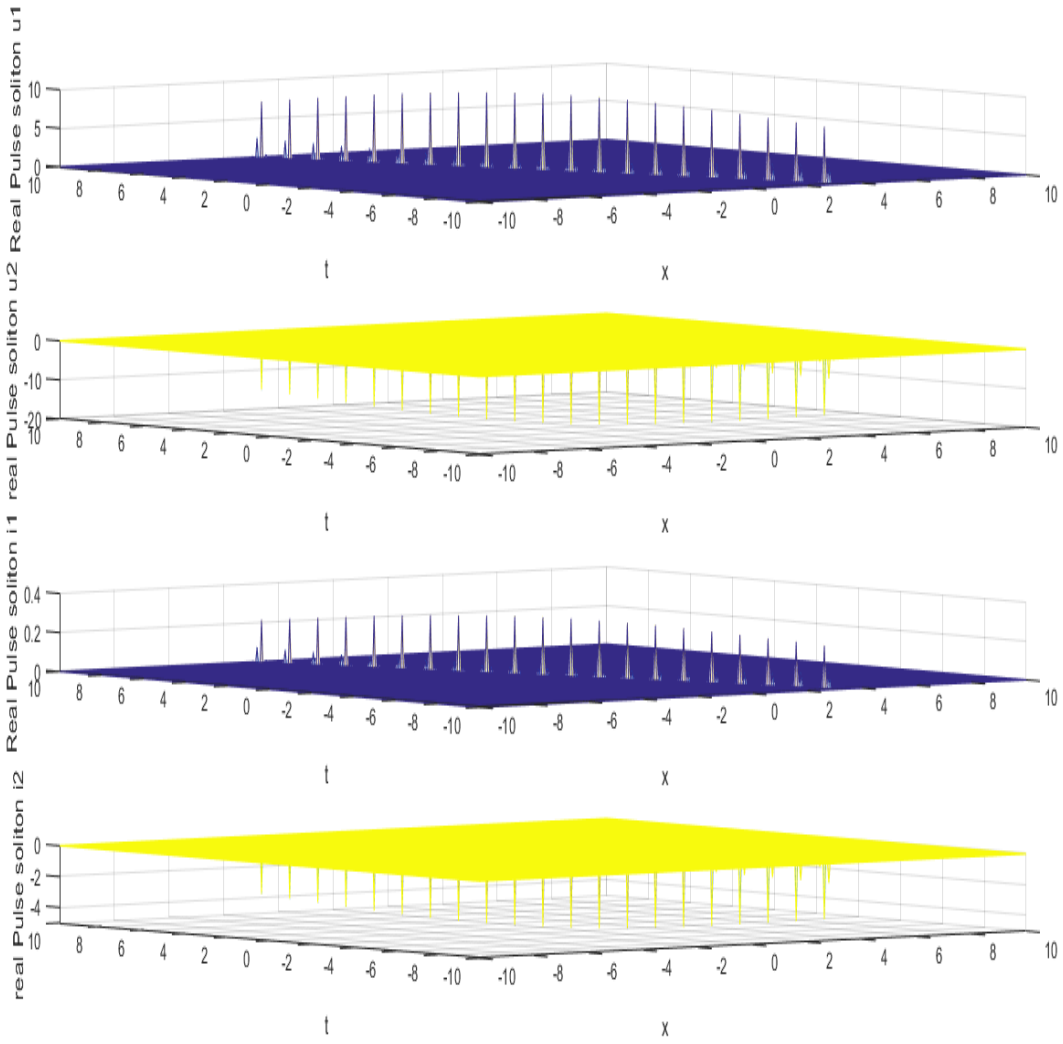

Figure 3: Real profile of the four Pulse solitons

This representation shows real profile of the four pulse solitons which are non-topological solitons since the properties of their media are the same at infinity.

5. Conclusion

The choice of nonlinear hybrid electrical line with crosslink capacitor for our study is due to the fact that it permits the simultaneous propagation of four signals contrary to a non-coupled hybrid electrical line which permits the simultaneous displacement of two signals; let us recall that the more we will multiply the crosslink capacitor in the line, the more we will multiply the

simultaneous movement of signals in the line. In the domain of physics in general and particularly in the domain of telecommunication, the set of four solitary waves obtained in (12) will permit the manufacturing of a new hybrid electrical line with crosslink capacitor where the flux linkage of its inductors and the charge of its capacitors vary in nonlinear manner defined in (7). In the same light, the set of four solitary waves obtained in (18) will permit the manufacturing of another hybrid electrical line with crosslink capacitor where the flux linkage of its inductors and the charge of its capacitors vary in nonlinear manner defined in (13). The set of four solitary waves obtained in (12) and in (18) prove that the quality of signals which are being propagated in the nonlinear hybrid electrical line with crosslink capacitor was ameliorated as compared to sinusoidal signals which are being propagate in the hybrid electrical line with crosslink capacitor. In order to bring up new ideas on the stability of the four sets of solitary waves obtained, it is necessary for us to study next their modulational instability before carrying out a practical exercise where we will experiment the applicability and the perfection of the two new hybrid electrical lines with crosslink capacitor.

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