Original Research Article

Modelling and allocation of vegetable crops using Mathematical Programming

Abstract

Mathematical programming techniques are commonly used by decision makers and achieving efficiency in agricultural production planning. Due to increasing demands of growing population of world, one needs to utilize the limited available resources in the most efficient and economic way. In this paper, the fractional programming problem is formulated and is used to determine the optimal cropping pattern of vegetable crops in such a way that the total profit is maximized. The solution of the formulated Fuzzy programming problem is obtained using LINGO.

Keywords: optimal solution, optimal land allocation, Fractional goal programming,

Multiobjective linear programming Problem.

Introduction:

In agricultural field experiments, crop planning is usually carried out to determine which type of crops should be cultivated and the area required for planting the crop. This planning issue is usually solved by using Mathematical programming techniques. Linear programming is one of the oldest techniques of Mathematical programming used for decision making studies. The most ordinary kind of mathematical programming is Fractional programming Romero and Rehman (1989) with ratio of objectives. Due to increasing demands of growing population of world the manufacture may have to invest a little more than the initial proposed budget in the interest of his production process. In this situation fuzzy set theory can be used to formulate the model with the help of membership functions. Most of the applications in agricultural planning correspond to the problem of determining an optimum-cropping pattern with multiple goals. Goal Programming techniques have been successfully used for these purposes Romero (1991). Multiobjective linear plus linear fractional programming problem solutions are found in Hirche (1984), Chadha (1993), Jain and Lachhwani (2008), Schaible (1977), Dangwal *et.al* (2012), Lone *et.al* (2015) etc. The first mathematical formulation of fuzziness was pioneered by Zadeh (1965). Orlovsky (1980) made a numerous attempts to explore the ability of fuzzy set theory to become a useful tool for adequate mathematical analysis of real world problems. Fuzzy methods have been developed in virtually all branches of decision making problems can be found in Tamiz (1996), Zimmermann (1991) , Kumari .*et.al*(2014) and Ross (1995). Goal programming approach in fuzzy environment has been first introduced by Narashimann (1980).Fuzzy goal programming has been discussed by several authors (see Pal *et al.* (2003) and Parra *et al.* (2001) , Lone *et.al*.(2016) etc,).

In this paper we have demonstrated that how a farmer who has limited resources such as availability of labor work time, water and land on which he/she wanted to grow three vegetable crops, Bringal, Tomato and ladies finger. The farmer's objective is to determine the optimal cropping pattern so that the total profit will be maximized

Linear Fractional Programming

A problem in which the objective function is the ratio of two linear functions and constraints are linear. Such problems are called linear fractional programming problems and can be stated precisely as fallows:

$$\begin{aligned}&Optimize\ Z=\frac{p'x+\alpha}{q'x+\beta}\\&subject\ to\\&\qquad Ax=b\\&\qquad x\geq 0\end{aligned}$$

where p and q are n vectors, b is an m vector. A is $m\times n$ matrix. α and β are scalars. If an optimal solution for a linear fractional problem exists, then an extreme point optimum exists.

Mathematical Formulation of General Multi-objective Programming Problem

The general multi-objective programming problem with n decision variables, m constraints and p objective is:

$$\left.\begin{aligned}Optimize\ Z&=Z(X_1,X_2,\ldots,X_n)\\&=[Z_1(X_1,X_2,\ldots,X_n),\\&\quad Z_2(X_1,X_2,\ldots,X_n),\\&\quad\ldots,\ Z_p(X_1,X_2,\ldots,X_n)]\end{aligned}\right\}\qquad (D)$$

$$\begin{aligned}&subject\ to\\&\qquad g_i(X_1,X_2,\ldots,X_n)\leq 0\\&and\quad X_j\geq 0,\quad (i=1,2,\ldots,m,\ j=1,2,\ldots,n)\end{aligned}$$

where, $Z(X_1,X_2,\ldots,X_n)$ is the multi-objective function with $Z_1(X_1,X_2,\ldots,X_n)$, $Z_2(X_1,X_2,\ldots,X_n),\ldots,\ Z_p(X_1,X_2,\ldots,X_n)$ are p individual objective functions.

For multi-objective linear programming problem (MOLPP) the proposed approach can be outlined as given below:

Step 1: solve problem with each single objective. Here P=3 and Find the minimum value of $MaxZ1, MaxZ2, and\ MaxZ3$, suppose $MaxZ2$, has minimum optimal value.

Step 2: Divide each objective individually say by $MaxZ2,$.

Step 3: we get fractional programming $\xi_1(z1(x)), and\ \xi_2(z2(x))$

Step 4: Define the membership function for Pth objective.

If $Z_p(x) \le g_p$ then

$$\mu_p(x) = \begin{cases} 1 & if \quad Z_p(x) \le g_{pt} \\ \dfrac{u_p - Z_p(x)}{u_p - g_p} & if \quad g_p \le Z_p(x) \le u_p \\ 0 & if \quad Z_p(x) \ge u_p \end{cases}$$

If $Z_p(x) \ge g_p$ then

$$\mu_t(x) = \begin{cases} 1 & if \quad Z_p(x) \ge g_p \\ \dfrac{Z_p(x) - l_p}{g_p - l_p} & if \quad l_p \le Z_p(x) \le g_p \\ 0 & if \quad Z_p(x) \le l_p \end{cases}$$

where g_p is the aspiration level of the tthobjective $Z_p(x)$ and u_p and l_p (p= 1, 2 . . . m) are the upper tolerance limit and lower tolerance limit, respectively, for the pth fuzzy goal. Zimmermann (1983) presented a fuzzy approach to multi-objective linear programming problems. Now, we formulate the fuzzy programming model of problem (D) by transforming the objective functions into fuzzy goals by assigning aspiration level to each of them using Zimmermann (1978) Max-min approach.

Step 5: Now, transform non linear membership functions $\mu_p(x)$ into an equivalent linear membership functions at individual best solution point by using first order Taylor's series as fallows:

$$\mu_p(x) = \mu_p(x_p^*) + [(x_1 - x_{p1}^*)\frac{d\mu_p(x_p^*)}{dx_1} + (x_2 - x_{p2}^*)\frac{d\mu_p(x_{pt}^*)}{dn_2} + . \ . \ . + (x_L - x_{tL}^*)\frac{d\mu_p(x_p^*)}{dx_L}]$$

where x_t^* is the individual best solution.

Step 6: Solve the fuzzy goal problem using LINGO.

Numerical Illustration

A farmer has 8 acres farm on which he/she grow three vegetable crops, Bringal, Tomato and ladies finger. As per his/her past his expense, the total available labor work time and total availability of water are 200 (000hrs hours) and 30(acre-inches) respectively. The information related to total profit in lakhs obtained from these three crops for one acre of land is given in the tabular form below. Now, the farmer's objective is to determine the optimal cropping pattern so that the total profit will be maximized. This example is a variant of Kumari *.et.al*(2014).

Vegetable crops	Profit (plot1)	Profit (plot2)	Profit (plot3)	Labor requirement (00hrs)	Water requirement/ acre-inches
Bringal	1.20	2.30	0.48	1.40	20.4
Tomato	0.35	0.45	1.15	1.30	17.5
Ladies finger	1.60	0.30	0.90	1.80	24.5

Let x1 be the area required in acres for Bringal crop.
Let x2 be the area required in acres for Tomato crop and
Let x2 be the area required in acres for ladies finger crop.
Therefore, the multi objective problem can be formulated as

$$MaxK1 = 1.20X_1 + 2.30X_2 + 0.48X_3$$

$$MaxK2 = 0.35X_1 + 0.45X_2 + 1.15X_3$$

$$MaxK3 = 1.60X_1 + 0.30X_2 + 0.90X_3$$

Subject to

$$X_1 + X_2 + X_3 \leq 8$$

$$1.40X_1 + 1.30X_2 + 1.80X_3 \leq 200$$

$$20.4X_1 + 17.5X_2 + 24.4X_3 \leq 30$$

Using step 1, we get

$$MaxK1 = 3.94, (0, 1.71, 0)$$

$$MaxK2 = 1.41, (0, 0, 1.23)$$

$$MaxK3 = 2.35, (1.47, 0, 0)$$

Using step (2 and 3), we have

$$Max\zeta_1$$

$$Max\zeta_2$$

Subject to

$$X_1 + X_2 + X_3 \leq 8$$

$$1.40X_1 + 1.30X_2 + 1.80X_3 \leq 200$$

$$20.4X_1 + 17.5X_2 + 24.4X_3 \leq 30$$

After solving this we get

$$Max\zeta_1 = 5.11, (0, 0.94, 0)$$

$$Max\zeta_2 = 4.57, (0.32, 0, 0)$$

It is observed that $Max\zeta_1 \geq 0, and\ Max\zeta_2 \geq 0$

Using step 4 and 5,we have

$$\zeta_1(x) = -1.39X_1 - 12X_3 + 5.11$$
$$\zeta_2(x) = -0.16X_1 - 0.154X_3 + 4.57$$

Thus the fractional programming problem is now transferred in Linear programming. The fuzzy goal programming is as follows with their possible aspiration levels as given below:

$$\zeta_1(x) = -1.39X_1 - 12X_3 + 5.11 \le 05.11$$
$$\zeta_2(x) = -0.16X_1 - 0.154X_3 + 4.57 \le 4.57$$
$$\textit{Subject to}$$
$$X_1 + X_2 + X_3 \le 8$$
$$1.40X_1 + 1.30X_2 + 1.80X_3 \le 200$$
$$20.4X_1 + 17.5X_2 + 24.4X_3 \le 30$$

Let (6, 5) be the tolerance limits for two goals respectively. The membership function can be defined for both of the two goals as

$$\mu_1(x) = \begin{cases} 1 & if \quad \zeta_1(x) \le 5.11 \\ \dfrac{6-\zeta_1(x)}{0.83} & if \quad 5.11 \le \zeta_1(x) \le 6 \\ 0 & if \ \zeta_1(x) \ge 6 \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & if \quad \zeta_2(x) \le 4.57 \\ \dfrac{5-\zeta_2(x)}{0.43} & if \quad 4.57 \le \zeta_2(x) \le 5 \\ 0 & if \ \zeta_2(x) \ge 5 \end{cases}$$

Now, using step 6, Fuzzy goal programming can be formulated as

$$Max\, G = \mu_1 + \mu_2$$
$$\textit{Subject to}$$
$$X_1 + X_2 + X_3 \le 8$$
$$1.40X_1 + 1.30X_2 + 1.80X_3 \le 200$$
$$20.4X_1 + 17.5X_2 + 24.4X_3 \le 30$$
$$\mu_1 \le 1$$
$$\mu_2 \le 1$$

The solution of the above problem can be obtained using LINGO. The optimal allocation is $X_1 = 0, X_2 = 0, and\ X_3 = 0.28$ X and the optimal maximized profit is 2(lakhs).

Conculsion

This study demonstrated the use of multiobjective linear programming problem for solving a production planning problem. It concludes that the formulated fuzzy fractional programming shows how the farmer obtained optimal cropping pattern which maximized total profit with the use of limited resources.

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