DEVELOPMENT OF RAINFALL INTENSITY - DURATION – FREQUENCY MODELS FOR AKURE, SOUTH-WEST, NIGERIA

ABSTRACT

 The rainfall Intensity-Duration-Frequency (IDF) relationship is widely used for adequate estimation of rainfall intensity over a particular catchment. A 25 year daily rainfall data were collected from Nigerian Meteorological Agency (NIMET) Abuja for Akure station. Twenty five year annual maximum rainfall amounts with durations of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300 and 420 minutes were extracted and subjected to frequency analysis using the excel solver software wizard. A total of six (6) return period specific and one (1) general IDF models were developed for return periods of 2, 5, 10, 25, 50 and 100 years using Gumbel Extreme Value Type I and Log Pearson Type III distributions. Anderson Darling goodness of fit test was used to ascertain the best fit probability 13 distribution. The R^2 values range from 0.982 to 0.985 for GEVT I and 0.978 to 0.989 for Log Pearson type III while the Mean Squared Error from 33.56 to 156.50 for GEVT I and 43.01 to 150.63 Log Pearson Type III distributions respectively. The probability distribution models are recommended for the prediction of rainfall intensities for Akure metropolis.

Keywords: IDF models, log Pearson Type III distributions, Excel Solver, goodness of fit test, Akure.

1. INTRODUCTION

 The Rainfall Intensity Duration Frequency (IDF) relationship is one of the most commonly used tools for the design of hydraulic and water resources engineering control structures. The IDF relationship is a mathematical relationship between the rainfall intensity, duration and the frequency (return period). The establishment of such relationship was done as early as 1932 (Bernard, 1932). The knowledge of frequency of extreme events like floods, high winds droughts and rainstorm helps in planning and design for these extreme events (Hosking and Wallis, 1997). The planning and designing of various water resources projects requires the use of rainfall intensity-duration-frequency (IDF) relationship (El-sayed, 2011). This relationship is determined through frequency analysis of data from meteorological stations. The IDF formulae are the empirical equations representing a relationship among maximum rainfall intensity (as dependent variable) and other parameters of interest such as rainfall duration and frequency (as independent variables). There are several commonly used functions found in the literature of hydrology applications (Chow et al., 1988). Owing to its wide applications, accurate estimation of intensity-duration-frequency relationship has received attention from researchers and scientists from all over the world (Mohammad Zakwan, 2016). All functions have been widely applied in hydrology. The IDF relation is mathematically stated as follows:

$$
I = f(T, d) \tag{1}
$$

 Where T is the return period (years) and d duration (minutes). Examples of three different types of empirical equations was documented by Nwaogazie & Masi (2019); Itolima & Nwaogazie (2017).

2. MATERIALS AND METHODS

2.1 Description of Area of Study

Akure is in Ondo State which is one of the States in Nigeria created on February 3, 1976 from the

42 former Western State. It lies within 7^0 10' N and 5^0 05' E. Akure is located in the rain forest of

 Nigeria. The available rainfall data (amount and duration) obtained from NIMET covered the period between 1986 and 2010.

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Figure 1: Location Map of Akure in South-Western Nigeria (Map data © 2019 Google)

Precipitation is characterized by a double maxima rainfall which starts from April and ends in

October, reaching its peak in June and September. The average annual rainfall is about 1,422mm with

some variations within the metropolis (analysed NIMET data).

2.2 Data Collection & Analysis

The major material used for this work is rainfall data comprising of rainfall amount and duration. The

twenty five (25) year rainfall data included data ranging from 1986 to 2010.The data were obtained

from Nigeria Meteorological Centre (NIMET) office Abuja, Nigeria. The data arrangement involved

sorting the mean data according to years, rainfall intensities and durations. The rainfall intensities

selected were the maximum values for each year for all the years analysed.

The annual maximum rainfall amount was obtained by selecting the maximum amount of rainfall for

each year for 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 (duration - minutes) for the 25

- years. Table1 shows the ranked observed annual maximum rainfall amounts for Akure.
- Table 1: Ranked Observed Annual Rainfall Amounts for Different Durations for Akure

 The rainfall amounts in Table 1were converted to intensity (mm/hr) by dividing the amount of rainfall by the duration then multiplying by 60. For instance given an amount of 70.3mm and duration of 15 minutes yields 281.3 mm/hr. Table 2 shows all the intensities for various durations.

 Table 2: Ranked Observed Annual Rainfall Intensities (mm/hr) for different Durations (mins) for Akure

18	144.6	102.6	84.8	68.0	58.0	54.4	44.9	35.3	27.5	21.4	16.1	12.9	9.2
19	143.6	100.0	82.4	64.6	51.9	46.8	44.3	34.9	26.2	21.4	16.0	12.8	9.2
20	141.7	99.4	82.4	63.6	49.3	45.5	43.9	33.7	25.3	20.1	15.1	12.1	8.9
21	136.6	99.1	80.8	63.0	48.2	44.5	40.8	33.1	24.1	19.8	14.8	11.9	8.6
22	135.9	99.0	78.3	62.6	48.1	43.1	39.3	32.1	22.0	19.0	14.3	11.4	8.5
23	135.6	96.2	76.4	62.4	47.8	39.6	38.8	29.3	20.9	18.9	14.2	11.4	8.5
24	130.3	91.2	75.8	61.8	47.7	37.6	35.4	27.2	20.6	17.4	13.0	11.2	8.5
25	127.9	91.1	75.7	60.6	46.2	36.7	32.7	26.5	20.4	15.7	13.0	10.8	8.3
Mean	181.2	138.3	116.9	96.7	77.9	67.1	57.2	41.9	32.5	24.7	18.8	15.1	11.2
Stand													
ard													
Deviat													
ion	59.0	43.2	35.3	32.8	28.5	21.2	15.7	10.2	8.1	6.6	4.9	3.9	2.9
Coeffi													
cient													
of													
Skewn													
ess	1.16	0.54	0.85	0.72	0.62	0.29	0.33	0.43	0.06	2.36	2.36	2.39	2.07

65 The magnitude of rainfall intensities were obtained using frequency analysis. Log Pearson Type III 66 distribution was used to obtain the magnitude of rainfall intensities for different return periods.

68 **2.3 Gumbel's Extreme Value Type I (GEVT- 1) Distribution**

69 Gumbel distribution is one commonly used probability distribution for obtaining the rainfall 70 intensity values. The rainfall intensity values were obtained using Equation (2) (Nwaogazie & Masi, 71 2019)

$$
X_T = \overline{X} + K_T S \tag{5}
$$

- 73 Where X_T = rainfall intensity values (magnitude of hydrologic event)
- 74 \bar{X} = mean; K_T = Gumbel's frequency factor; S = standard deviation
- 75 The Gumbel's frequency factor is obtained using Equation (3).

76
$$
K_T = -\frac{\sqrt{6}}{\pi} \left\{ 0.5772 + In \left[In \left(\frac{T}{T-1} \right) \right] \right\}
$$
 (6)

77 Where $T =$ return period (years)

78 For example, Gumbel frequency factor for a 5 years return period

79
$$
K_T = \frac{\sqrt{6}}{\pi} \left\{ 0.5772 + In \left[In \left(\frac{5}{5-1} \right) \right] \right\} = 0.719
$$

80 The resulting Gumbel K_T values for different return periods as calculated are shown in Table 3.

81

82 Table 3: Gumbel frequency factor for Akure IDF modeling

2.4 Log Pearson Type III (LPT III) Distribution

 Log Pearson type III distribution is one commonly used probability distribution for obtaining the rainfall intensity values. The rainfall intensity values were obtained using Equation (5)

$$
Log X_T = Log \overline{X} + K_T Log S \tag{7}
$$

88 Where X_T = rainfall intensity values (magnitude of hydrologic event)

89
$$
\bar{X}
$$
 = mean; K_T = Log Pearson frequency factor; S = standard deviation

 Log-Pearson frequency factor can be obtained from the frequency table given in standard textbook using the return period and the skewness from Table 3 as follows:

 For example, Log-Pearson distribution frequency factor for a 10 mins duration and 5 years return period with coefficient of skewness = 0.366734 was calculated to be 0.81866.

95 Table 4 gives the computed summary of K_T values for Log-Pearson distribution for various durations and different return periods computed.

Table 4: Log-Pearson frequency factors for various durations and return periods

Calibration of Sherman (1932) IDF model

Sherman's modified IDF model is given as (Chow et al, 1988)

$$
101 \t I = \frac{c T_r^m}{T_d^a} \t (6)
$$

 Equation (6) is non-linear power law that was calibrated for c, m, a parameters using intensity, duration and return period values in Table 1 and Excel Optimization Solver (Nwaogazie & Masi, 2019).

2.5 Goodness of fit test

The result in Table 1 was subjected to Anderson-Darling test to ascertain the probability distribution

- that best fit the rainfall annual maximum amount. This is a nonparametric test of the equality of
- continuous, one dimensional probability distributions that can be used to compare a sample with a
- reference probability distribution. Gumbel Extreme Value Type I (GEVT-1) and Log-Pearson Type 3
- (LPT-3) best fit the rainfall intensities with significant values of 0.7570 and 0.7538 at 5% confidence
- level respectively.

3. RESULTS

3.1 Development of Intensity Duration Frequency (IDF) Models

 Figure 2 represents the rainfall intensity values for various durations for the different return periods using Gumbel Extreme Value Type I distribution.

 Figure 2: Intensity Duration Curve (IDF) curves for Gumbel Extreme Type 1 distribution plotting 119 Intensities (mm/hr) against durations (mins) for Akure

- 120 121 The intensity duration frequency models were calibrated using the Microsoft Excel Solver. The 122 method adopted uses the least square criteria to obtain the model parameters. Table 4 gives a 123 distribution of developed IDF models for Gumbel Extreme Value Type 1 distribution.
- 124 Table 5 gives a distribution of developed IDF models for Gumbel Extreme Value Type 1 distribution.
- 125
- 126 Table 5: Developed IDF Models for different return periods using Gumbel Extreme Value Type 1 127 distribution rainfall intensities values for Akure.
- 128

129 **±** return period specific IDF models

130 The general IDF models was developed using Excel Spread Sheet Solver tool. The least

131 squares equations were programmed accordingly.

132
$$
I = \frac{407.886T_r^{0.175}}{T_d^{0.525}}
$$
 (7)

133 We note the following results: coefficient of determinant $(R^2) = 0.982$; Mean Squared Error = 125.70

134 mm/hr

135

136 **3.2 Development of Intensity Duration Frequency (IDF) Models**

137 Figure 3 shows the rainfall intensity values for various durations for the different return periods using 138 Log Pearson Type III distribution.

141 Figure 3: Intensity Duration Frequency (IDF) curves for Log Pearson Type III distribution for Akure.

142 The intensity duration frequency models were developed using the Microsoft Excel Solver. The 143 method employs the least square criteria to obtain the model parameters.

144 Table 6 gives a distribution of developed IDF models for Log Pearson Type III distribution for Akure

145

148

146 Table 6: Developed IDF Models for different return periods using Log Pearson Type III distribution 147 rainfall intensities values for Akure.

A general IDF model was also developed. This model enables one to predict the intensity of rainfall of

any duration and any return period.

$$
153\,
$$

$$
I = \frac{402.607 T_r^{0.201}}{T_d^{0.540}}
$$
 (8)

154 We note the following results: coefficient of determinant $(R^2) = 0.984$; and Mean Squared Error = 127.47

Excel Solver Log Pearson Type III model parameters trial solution for 5 year return

period specific IDF model has eleven (11) iterations before convergence (see Table 7).

Table 7: Excel Solver iteration distribution to convergence

3.2 Comparison of Observed and Predicted Rainfall Intensity

The intensity duration frequency curves were obtained by plotting the predicted rainfall

 intensity values against corresponding durations for different return periods. The IDF curves for Akure are as shown in Figures 4 - 6.

 Figure 5: Observed rainfall intensity compared with predicted for 5 and 25 year return period for Log-Pearson Type-3 distribution for Akure

 3.3 Comparison of Regression Approach and Excel Optimization Solver results for model parameters, 196 \overline{R}^2 and MSE

 Table 8 (an extension of Table 5) clearly shows the result from Excel Optimization Solver option is superior to the normal regression method, the conventional simultaneous solution using matrix i.e. Gauss elimination, inverse or determinant approach (Nwaogazie & Masi, 2019).

 Table 8: Results from regression approach and excel solver optimization approach (Log Pearson Type III, 2 year return period)

MSE

Method C m A R^2 Regression 65.52 3.544 0.675 0.885 324.40 Excel Solver | 4.74 | 6.366 | 0.500 | 0.980 | 43.01

4. CONCLUSION

 The developed model for Log Pearson Type III is in agreement with literature theory which shows higher intensity occurring at lower duration and lower intensity at higher duration. The prediction of rainfall intensity with the PDFs showed a good match with observed intensity values. The log Pearson Type III model ranked as the best with 211 respect to MSE 43.01 and R^2 0.989 in the return period specific model when compared 212 with GEVT-I with MSE 33.56 and R^2 0.985. The comparison of PDF and non-PDFs shows that the former has lesser MSE value than the later; 43.01 and 324.40 respectively.

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