RAINFALL INTENSITY-DURATION-FREQUENCY MODELS USING OPTIMIZATION TECHNIQUE FOR ABEOKUTA, SOUTH – WEST NIGERIA

ABSTRACT

The design of water resources engineering control structures is best achieved with adequate estimation of rainfall intensity over a particular catchment. To develop the rainfall intensity, duration and frequency (IDF) models, 25 year daily rainfall data were collected from Nigerian Meteorological Agency (NIMET) Abuja for Abeokuta. The annual maximum rainfall amounts with durations of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300 and 420 minutes were extracted and subjected to frequency analysis using the Excel Optimization Solver wizard. Specific and general IDF models were developed for return periods of 2, 5, 10, 25, 50 and 100 years using the Gumbel Extreme Value Type 1 and Log Pearson Type 3 distributions. The Anderson Darling goodness of fit test was used to ascertain the best fit probability distribution. The R^2 values range from 0.973 – 0.993 and the Mean Squared Error, MSE from 84.49 – 134.56 for the Gumbel and 0.964 – 0.997 with MSE of 42.88 – 118.68 for Log Pearson Type 3 distribution, respectively. The probability distribution models are recommended for the prediction of rainfall intensities for Abeokuta metropolis.

Keywords: IDF models, Gumbel Extreme Value Type I, Log Pearson Type 3 distributions, Excel Optimization Solver, goodness of fit test.

1. INTRODUCTION

The Rainfall Intensity Duration Frequency (IDF) relationship is one of the most commonly used tools for the design of hydraulic and water resources engineering control structures. An IDF model is a mathematical relationship between the rainfall intensity, duration and the frequency (return period). The establishment of such relationship was done as early as 1932 (Bernard, 1932). The knowledge of frequency of extreme events like floods, droughts, rainstorm and high winds assisted in planning and design for these extreme events (Hosking and Wallis, 1997). The planning and designing of various water resource projects requires the use of rainfall intensity-duration-frequency (IDF) relationship (El-sayed, 2011). This relationship is determined through frequency analysis of data from meteorological stations. The IDF formulae are the empirical equations representing a relationship among maximum rainfall intensity (as dependent variable) and other parameters of interest such as rainfall duration and frequency (as independent variables). There are several commonly used functions found in the literature of hydrology applications (Chow et al., 1988). Owing to its wide applications, accurate estimation of intensity-duration-frequency relationship has received attention from researchers and scientists from all over the world (Mohammad Zakwan, 2016). All functions have been widely applied in hydrology. In Nigeria, a lot of work has been done in South – East and South – South like the IDF models of Port Harcourt (Nwaogazie & Duru, 2002) and that of Eket in Awka Ibom State

(Nwaogazie & Uba, 2001). All these models generated IDF curves that confirms the theory for shorter recurrence periods of 2 to 10 years.

2. MATERIALS AND METHODS

2.1 Area of Study

Abeokuta is the capital of Ogun State in South – West Nigeria covering an estimated area of about 40.60 km^2. It is located at 74m above the sea level and falls within latitude 7^o 10´N and 7^o 15´N and longitudes 3^o 17´E and 3^o 26´E. Abeokuta lies in the plane which is developed on rocks of the basement complex found in the Savannah zone. The area is properly drained by River Ogun and it is characterized by relatively high temperature with mean annual temperature of 30^oC and rainfall of 1,185 mm respectively.

Figure 1: Location map of Abeokuta and adjoining cities in South-Western Nigeria
Source: Google map (2019)

2.2 Data Collection

The major material used for this work is rainfall data comprising of rainfall amount and duration. The twenty five (25) years rainfall data included data ranging from 1986 to 2010.The data were obtained from Nigeria Meteorological Centre (NIMET) office Abuja, Nigeria. This data arrangement involved sorting the data according to years, rainfall intensities and durations. The rainfall intensities selected are the maximum values for each year for all the years analysed.

2.3 Data Analysis

The annual maximum data series are obtained by selecting the maximum amount of rainfall for each year for 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 durations (minutes) for the 25 year period.

The IDF relation is mathematically expressed as follows:

$$I = f(T,d) \quad (1)$$

The rainfall amount is converted to intensity (mm/hr) by dividing the amount by the duration (minutes) then multiplying by 60 as a conversion factor. For instance, given rainfall amount of 54.3mm for 15 minute duration yields an intensity of (54.3/15) x 60 = 217.2 mm/hr

Table 1 shows all the intensities for various durations.

Table 1: Ranked Observed Annual Rainfall Intensities (mm/hr) for different Durations (mins) for Abeokuta

Year	Rainfall intensity (mm/hr)												
	5	10	15	20	30	45	60	90	120	180	240	300	420
1	421.2	271.2	217.2	186.3	140.6	112.4	88.6	59.8	54.2	40.9	32.1	25.7	18.3
2	381.6	270.0	189.6	174.3	129.6	93.7	84.3	59.5	44.7	36.1	30.7	24.6	17.5
3	336.0	257.4	180.8	166.8	129.2	89.6	82.3	59.1	44.3	30.7	27.1	21.7	15.5
4	330.0	248.4	180.0	162.9	125.6	86.5	70.3	58.7	44.1	29.8	25.6	20.6	14.7
5	295.2	231.0	178.8	142.2	124.2	86.4	67.2	54.9	41.2	29.5	23.1	20.5	14.6
6	289.2	221.4	171.6	135.6	116.2	86.1	64.9	54.7	41.0	27.4	22.3	18.4	13.2
7	233.1	210.6	169.2	135.0	94.8	85.5	64.8	44.8	33.6	27.3	22.2	17.9	12.8
8	223.1	190.8	167.2	134.1	90.4	85.3	64.6	43.9	33.0	22.4	20.6	17.7	12.7
9	196.8	171.0	165.6	128.7	89.4	84.9	64.0	43.2	32.4	22.0	20.5	16.5	12.2
10	195.6	168.0	154.0	126.9	85.8	82.8	63.7	43.1	32.3	21.6	19.7	16.4	11.8
11	187.2	165.0	147.6	125.4	83.6	78.3	62.1	42.7	32.0	21.5	17.7	15.2	11.7
12	186.1	152.4	140.4	124.2	82.8	77.6	58.7	42.5	31.9	21.4	16.9	14.9	11.6
13	181.2	147.6	131.2	123.0	82.0	63.2	58.2	39.1	29.4	21.3	16.5	14.6	10.8
14	170.4	146.9	127.2	122.4	81.6	60.3	47.4	38.8	29.1	21.2	16.2	13.2	10.7
15	167.5	144.6	120.4	115.5	77.0	57.2	44.5	37.1	28.0	20.5	16.2	13.0	10.3
16	162.3	140.6	112.1	110.7	73.8	55.7	44.1	35.9	27.8	19.6	16.0	12.9	9.8
17	161.0	124.0	112.0	95.4	70.6	55.2	42.9	33.9	26.9	19.6	15.9	12.8	9.7
18	149.5	117.9	107.3	92.5	67.6	54.7	42.6	32.9	26.8	19.4	15.6	12.8	9.4
19	149.0	117.2	96.4	90.3	63.6	53.9	41.8	32.5	24.7	18.1	14.9	12.7	9.3
20	137.9	111.6	94.6	88.6	60.2	51.6	41.0	31.6	23.7	17.9	14.7	12.5	9.2
21	135.6	105.5	90.0	78.1	59.6	51.3	38.5	29.1	23.7	17.2	14.7	12.3	9.1
22	119.7	102.2	89.5	74.3	56.7	45.5	38.0	28.7	22.5	17.1	14.6	12.2	8.9
23	117.7	101.4	80.5	73.8	56.4	43.3	37.5	28.6	22.4	16.5	14.2	12.1	8.8

24	117.4	98.4	78.0	72.3	50.7	43.0	35.7	28.4	21.8	15.8	14.1	11.7	8.7
25	115.8	98.4	77.4	66.5	49.1	41.3	35.5	27.3	21.5	15.5	13.5	11.7	8.5
Mean	206.4	164.5	135.1	117.8	85.6	69.0	55.3	41.2	31.7	22.8	19.0	15.8	11.6
Standard Deviation	86.8	57.5	40.7	33.6	27.3	19.7	16.1	10.9	8.7	6.5	5.3	4.1	2.8
Coefficient of Skewness	1.05	0.63	0.16	0.25	0.64	0.26	0.51	0.53	0.95	1.32	1.19	1.08	1.01

The magnitude of rainfall intensities were obtained using frequency analysis. Two probability distributions namely Gumbel Extreme Value Type I (GEVT-1) and Log-Pearson Type III were used to obtain the magnitude of rainfall intensities for different return periods.

2.4 Gumbel's Extreme Value Type I (GEVT- 1) Distribution

Gumbel distribution is one commonly used probability distribution for obtaining the rainfall intensity values. The rainfall intensity values were obtained using Equation (2)

$$X_T = \bar{X} + K_T S \qquad (2)$$

Where X_T = rainfall intensity values (magnitude of hydrologic event)

$\bar{X}$ = mean; K_T = Gumbel's frequency factor; S = standard deviation

The Gumbel's frequency factor is obtained using Equation (3).

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + In\left[In\left(\frac{T}{T-1}\right)\right]\right\} \qquad (3)$$

Where T = return period (years)

For example, Gumbel frequency factor for a 5 years return period

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + In\left[In\left(\frac{5}{5-1}\right)\right]\right\} = 0.719$$

The resulting Gumbel K_T values for different return periods as calculated are shown in Table 2.

Table 2: Gumbel frequency factor for Abeokuta IDF modeling

Return Period	2	5	10	25	50	100
K_T values	-0.16425	0.719	1.304	2.044	2.592	3.1363

Calibration of Sherman (1932) IDF model

Sherman's (1932) IDF model is given as

$$I = \frac{C T_r{}^m}{T_d{}^a} \qquad (4)$$

Equation (4) is non-linear power law that was calibrated for c, m, a parameters using intensity, duration and return period values in Table 1 and Excel Optimization Solver.

Goodness of fit test

The result in Table 1 was subjected to Anderson-Darling test to ascertain the probability distribution that best fit the rainfall annual maximum amount. This is a nonparametric test of the equality of continuous, one dimensional probability distributions that can be used to compare a sample with a reference probability distribution. GEVT-1 and Log-Pearson Type 3 (LPT-3) best fit the rainfall intensities with significant values of 0.7570 and 0.7538 at 5% confidence level respectively.

RESULTS

The Anderson-Darling test shows that GEVT-1 and log Pearson Type III best fit the rainfall annual maximum amounts as shown in Table 3

The rainfall intensity values are computed by applying Equation (1). Rainfall intensity using GEVT-1 distribution with the mean and standard deviation are obtained from Table 1 For a 5 minute duration and 2 years return period, the probability equivalent of rainfall intensity via GEVT-1 is $X_T = \bar{X} + K_T S$ ➢ $X_T = 200.3 + (-0.16425 \times 147.52))$ ➢ $X_T = 200.3 - 24.23$ ➢ $X_T = 176.07$mm/hr

Figure 2 shows rainfall intensity distributions and return periods using GEVT-1 distribution.

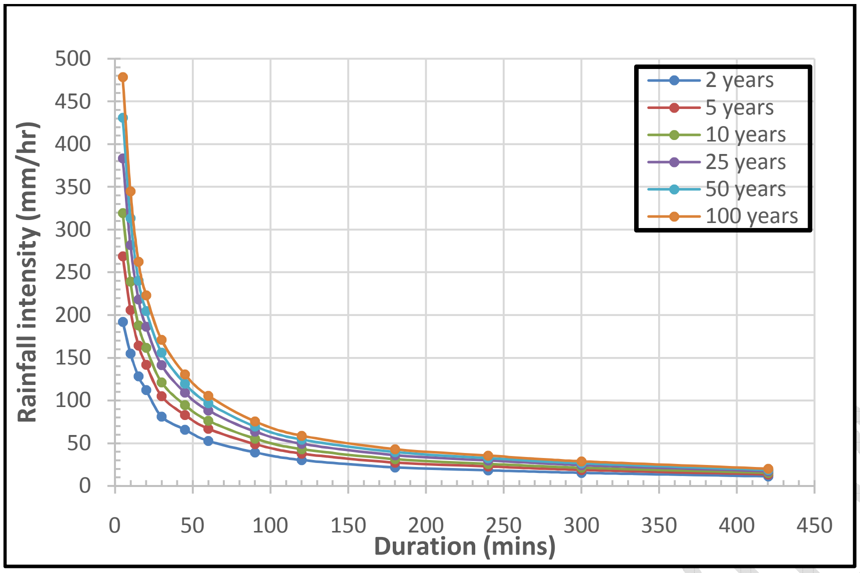

Figure 2 Intensity Duration Frequency (IDF) curves for GEVT - 1 distribution for Abeokuta.

Calibration of Sherman's IDF models for specific Return periods

The calibrated Sherman (1932) IDF models for specified return periods are as presented in Table 3. Equally included in the table are coefficient of determination R^2 and mean square error (MSE) for model performance assessment.

Table 3: GEVT-1 calibrated IDF Models for different return periods for Abeokuta.

Return Period	IDF Model ±	Coefficient of Determination (R^2)	Mean Squared Error (MSE)
2	$\mathrm{I} = \frac{4.857T_r^{\,6.663}}{T_d^{\,0.535}}$	0.973	84.49
5	$\mathrm{I} = \frac{2.2431T_r^{\,3.579}}{T_d^{\,0.569}}$	0.985	93.05
10	$\mathrm{I} = \frac{1.676T_r^{\,2.710}}{T_d^{\,0.584}}$	0.988	100.93
25	$\mathrm{I} = \frac{1.319T_r^{\,2.075}}{T_d^{\,0.599}}$	0.990	112.96

50	$I = \frac{1.197 T_r^{1.765}}{T_d^{0.608}}$	0.992	123.26
100	$I = \frac{1.113 T_r^{1.540}}{T_d^{0.615}}$	0.993	134.56

±: return period specific IDF models

Evaluation of iterative Equation Solver in Excel

Excel Solver model parameters trial solution for return period (2 year) specific IDF model has fourteen (14) iterations before convergence (see Table 4). Similarly, there are thirty-five (35) iterations in the development of the general IDF model given in Equation (6).

Table 4: Trial solution result for Sherman's specific IDF model calibration

Iteration	C	m	A
1	1	1	1
2	1.461474	1.31987	0
3	3.546129	3.431661	0
4	3.825354	4.117993	0
5	3.830287	4.130401	0.05
6	4.528795	5.887498	0.312129
7	4.713106	6.348498	0.400196
8	4.838772	6.614912	0.52986
9	4.859924	6.669481	0.538164
10	4.857193	6.663613	0.535575
11	4.856903	6.662889	0.535429
12	4.856903	6.662889	0.535429
13	4.856903	6.662889	0.535429
14	4.856903	6.662889	0.535429

The coefficient of determination is computed from Equation (5) and Table 5

$$R^2 = \frac{\left(\sum_{i=1}^{n}(y - {y_{avg}}^2) - \sum_{i=1}^{n}(y - y_{pred})^2\right)}{\sum_{i=1}^{n}(y - y_{avg})^2} \quad (5)$$

$$R^2 = \frac{(41807.74 - 1098.365)}{41807.74} = 0.973$$

Calculating the Mean Square Error (MSE) using Equation (6) we have;

$$\text{MSE} = \frac{\sum_{i=1}^{n}(y - y_{pred})^2}{n} \quad (6)$$

$$\text{MSE} = \frac{1098.365}{13} = 84.49$$

Table 5 Tabular Computation of Coefficient of Determination for 2 year return period

Intensity	Intensity_{pred}	(I - Ip)2	(I-Iavg)2
192.1498641	207.892929	247.8440829	14668.11
155.0966423	143.436046	135.9695073	7065.876
128.463877	115.444493	169.5043489	3297.745
112.3163251	98.9639205	178.2867085	1703.91
81.16415026	79.6511058	2.28930367	102.5414
65.78223051	64.1071879	2.805767634	27.62183
52.68677814	54.9554029	5.146658379	336.7629
39.42640188	44.2308529	23.08274969	999.2854
30.27733462	37.9165648	58.35783719	1661.422
21.74873497	30.517145	76.88501435	2429.42
18.13831768	26.1605922	64.35688805	2798.363
15.11094943	23.2144685	65.66702178	3127.821
11.13080687	19.3872836	68.16940809	3588.857
Average = 71.038		**Sum = 1098.365**	**Sum = 41807.74**

A general IDF model was also developed. A total of 13 durations multiplied by 6 return periods yields 78 input data point. The entire input data were taken from Table 1.

The general IDF model was developed using Excel Optimization Solver. The least squares equations were programmed accordingly.

$$I = \frac{551.809 T_r^{0.188}}{T_d^{0.596}} \qquad (6)$$

Coefficient of determinant (R^2) = 0.987; Mean Squared Error = 147.70 mm/hr

The plot of the predicted intensity values of Equation (6) is as given in Figure 3.

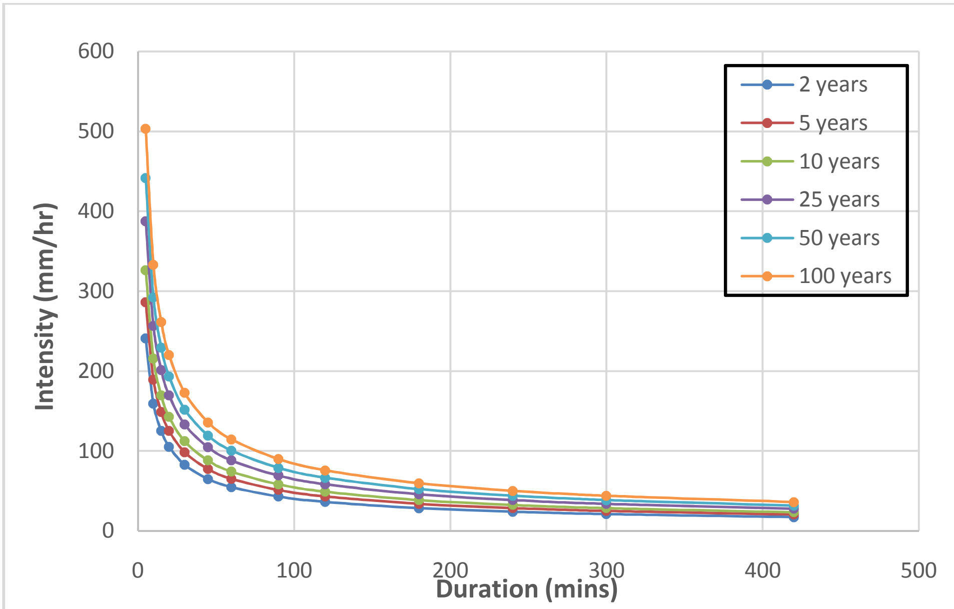

Figure 3 Intensity Duration Curve for Gumbel Extreme Value Type I IDF general model for Abeokuta.

Comparison of Observed and Predicted Rainfall Intensities

This model enables one to predict the intensity of rainfall of any duration and any return period. The verification of the developed model is carried out by plotting the observed and predicted intensities on the same graph as shown in Figures 4 to 6.

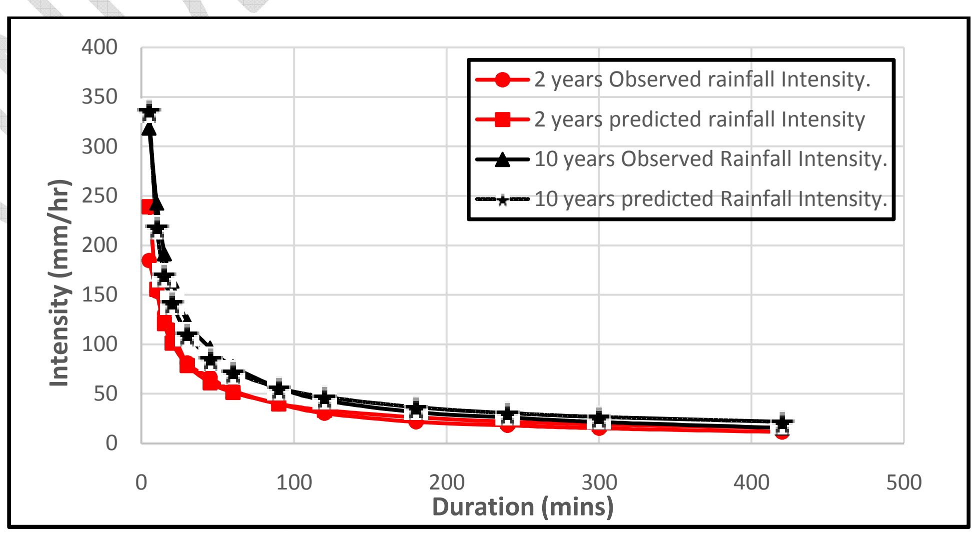

Figure 4 Observed rainfall intensity against predicted rainfall intensity for 2 and 10 year return periods for Log-Pearson Type-3 distribution

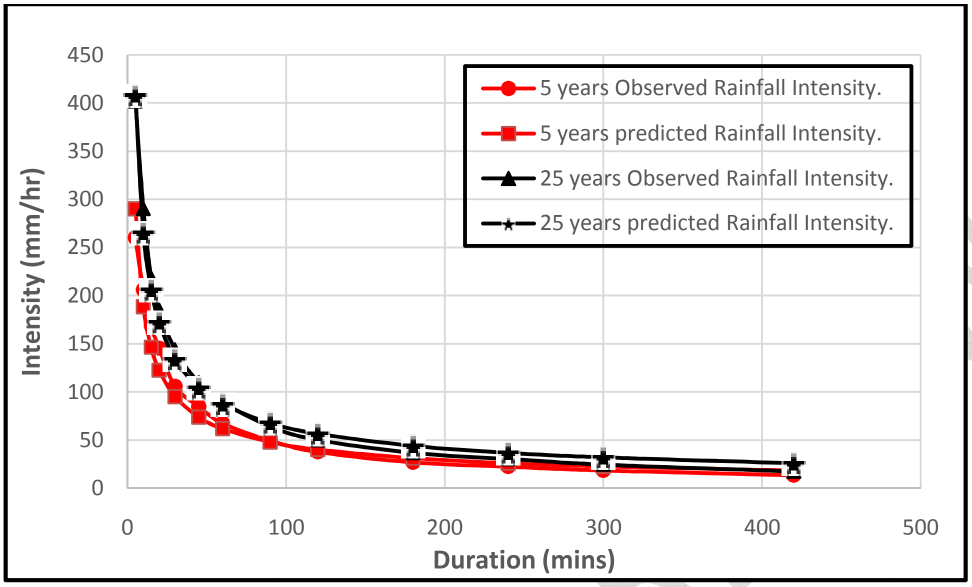

Figure 5 Observed rainfall intensity against predicted rainfall intensity for 5 and 25 year return periods for Log-Pearson Type-3 distribution

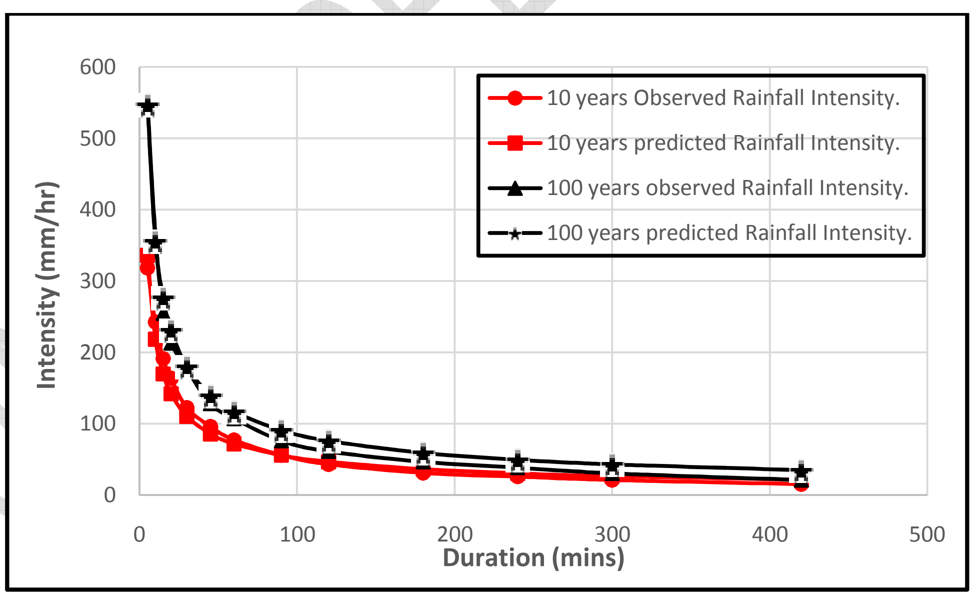

Figure 6 Observed rainfall intensity against predicted rainfall intensity for 10 and 100 year return periods for Log-Pearson Type-3 distribution

Comparison of Regression Approach and Excel Optimization Solver results for model parameters, R^2 and MSE

Table 6 (an extension of Table 5) clearly shows the result from Excel Optimization Solver option is more reliable than the normal regression method, the conventional simultaneous solution using matrix i.e. Gauss elimination, inverse or determinant approach.

Table 6 Results from regression approach and excel solver optimization approach (GEVT-1, 2 year return period)

Method	C	m	A	R^2	MSE
Regression	65.31	3.532	0.675	0.897	330.18
Excel	4.857	6.663	0.535	0.973	84.49

CONCLUSION

The developed models for GEVT-1 and Log Pearson Type III are in agreement with PDF theory which shows higher intensity occurring at lower duration and lower intensity at higher duration. The prediction of rainfall intensity with the PDFs showed a good match with observed intensity values. The log Pearson Type III model ranked as the best with respect to MSE 54.22 and R2 0.998 in the return period specific model. The comparison of PDF and non-PDFs shows that the former has lesser MSE value than the later; 84.49 and 330.18 respectively.

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