# Analysis of Transmission Dynamics of Anthrax in Animal Population. A modeling Approach


## Abstract

This paper seeks to develop a SIR model with vaccination compartment in the study of anthrax transmission dynamics in animal population. The model employ ordinary differential equations in the formulation of the model's equation. The model's steady states solutions are investigated. The disease free equilibrium and endemic equilibrium of the model are analyzed qualitatively. Vaccination rate below a certain critical value causes the anthrax disease to persist. Recruitment and contact rates are the most sensitive parameters that contribute significantly to the basic reproductive ratio.


## 1.0 Introduction

Anthrax is an infectious disease categorized under zoonotic diseases caused by a bacterium called *Bacillus anthracis* [4]. The disease is found naturally in soil [19] and mostly affects herbivores. Anthrax is one of the major diseases that cause uncontrolled mortality in cattle, pigs, sheep, goats and horses worldwide [3, 10, 15]. Animals easily get infected with anthrax through contact with infected animals, consumption of infected grass or water and by inhalation of anthrax spores [18]. The environment is usually infected with carcasses from infected animals. Grass and soil become the reservoirs of anthrax spores which can persist in the soil or grass for an extended period of time even under very extreme weather and environmental conditions.

Authors [1] model consists of susceptible, contamination, infective and pathogens. The model regards infective compartment as key to the transmission of anthrax. In this model, the infected animals do show clinical signs of the disease. According to author [2], the model consists of susceptible, contamination and pathogens. The model does not include infective compartment. According to the model, the infective compartment do have very low reproductive ratio [11] and does not cause any infections in animals.

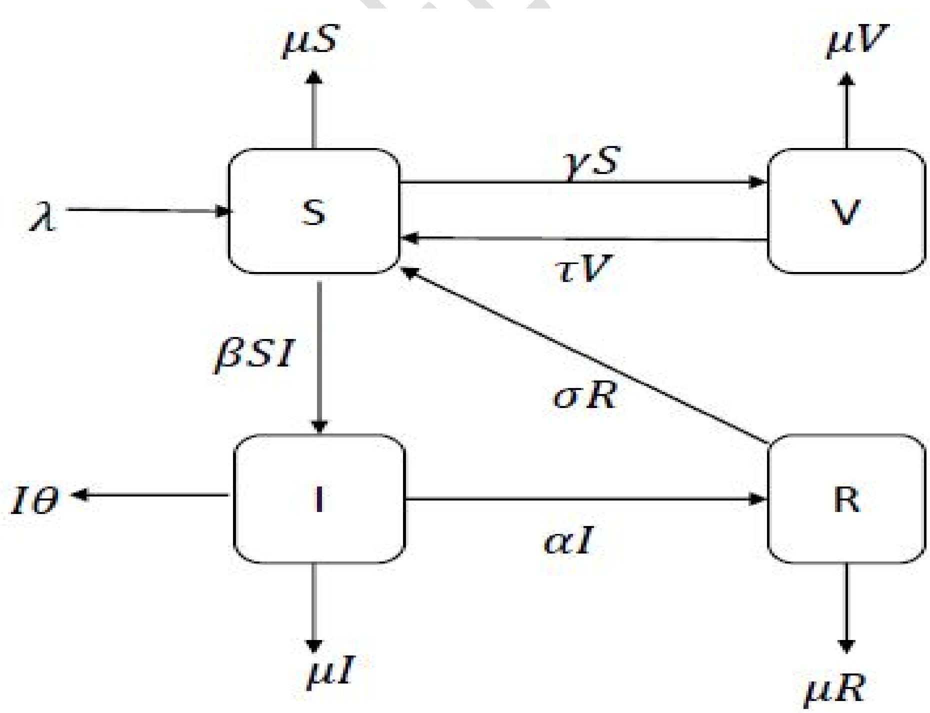
Research done by authors [8] the model considers transmission, carcass ingestion, environment and migration as possible means through which anthrax is transmitted in animals. In this model, carcass ingestion and removal of carcasses from the environment does not cause any decline of anthrax transmission in animals.

## 2.0 The Model

This model divides the total animal population at any time (t) into four sub compartments with respect to their disease status in the system. The total animal population is given by N (t) =S(t)+I(t)+R(t)+V(t) where S (t) represents animals at risk of developing anthrax infection, I (t) represents animals showing anthrax symptoms, R (t) represents animals recovered from anthrax infection and acquired temporal immunity and V (t) represents animals susceptible and are vaccinated against anthrax attack.

The parameters used in this model are: $\lambda$ denotes recruitment rate; $\beta$ denotes contact rate; $\mu$ denotes natural death rate; $\gamma$ denotes vaccination rate; $\tau$ denotes waning immunity of vaccinated animals; $\sigma$ denotes waning recovery rate; $\theta$ denotes disease induced death rate and $\alpha$ animal recovery rate.

The diagram below shows SIR model flow chart with vaccination compartment for anthrax transmission in animal population.




. Figure 1: SIR Flow chart with vaccination compartment

The model equations are:

$$\left.\begin{aligned} \frac{dS}{dt} &= \lambda - \beta SI - (\mu+\gamma)S + \sigma R + \tau V \\ \frac{dI}{dt} &= \beta SI - (\mu+\theta+\alpha)I \\ \frac{dR}{dt} &= \alpha I - (\mu+\sigma)R \\ \frac{dV}{dt} &= \gamma S - (\mu+\tau)V \end{aligned}\right\} (1)$$

Disease Free Equilibrium is given by $\varepsilon^0 = (S^0, I^0, R^0, V^0)$ .There exists no anthrax disease and no animals are infected with anthrax. The critical point is given by $\varepsilon^0 = (\frac{\lambda}{\mu+\gamma}, 0, 0, 0)$.

According to authors [3, 11], reproductive ratio can be found using Jacobian matrix J of (1) as:

$$J(SIRV) = \begin{pmatrix} -(\mu+\gamma) & -\beta\frac{\lambda}{\mu+\gamma} & \sigma & \tau \\ 0 & \beta\frac{\lambda}{\mu+\gamma} - (\mu+\theta+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & 0 & -(\mu+\tau) \end{pmatrix}$$

(2)

Determinant of (2) become $1 - \beta\frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}$

(3)

$$\beta\frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}$$

(4) Expression (4) is called the basic reproductive ratio $R_O$.

**Theorem 1**

Disease free equilibrium point is locally asymptotically stable if $R_0 < 1$ and anthrax disease will not persist.

If $R_0$ > 1, disease free equilibrium become unstable.

**Proof**

Disease free equilibrium point $\varepsilon^{\circ}(\frac{\lambda}{\mu+\gamma}, 0, 0, 0)$ has reproductive ratio given as

$$R_0 = \beta \frac{\lambda}{\mu+\gamma} - (\mu+\theta+\alpha)$$

At disease free equilibrium $\beta \frac{\lambda}{\mu+\gamma} - (\mu+\theta+\alpha) < 0$

(5)

Equation (5) can be expressed as $\beta \frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)} - 1 < 0$.

$$R_0 = \beta \frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}$$

Therefore, $R_0 - 1 < 0$ which implies that $R_0 < 1$.

Given that $R_0 < 1$, we have disease free equilibrium point which is locally asymptotically stable.

**Lemma 1**

If $\beta \frac{\lambda}{(\mu+\gamma)} - (\mu+\theta+\alpha) > 0$, then it follows that $\beta \frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)} - 1 > 0$

Therefore, $R_0 > 1$ which implies that the disease will persist in animal population.

According to authors in [15], endemic equilibrium of dynamical system (1) is given by $\varepsilon^* = (S^*, I,^* R,^* V^*)$ where $S^* > 0, I^* > 0, R^* > 0$ and $V^* > 0$.

From (2), the rest point becomes:

$$S^* = \frac{\mu+\theta+\alpha}{\beta} \quad ,$$

$$I^* = \frac{(\mu+\gamma)(\mu+\tau)(\mu+\theta+\alpha)-\gamma\tau(\mu+\theta+\alpha)-\beta\lambda(\mu+\sigma)}{\beta(\mu+\tau)\left[\sigma\tau-(\mu+\sigma)(\mu+\theta+\alpha)\right]}(\mu+\sigma)$$

$$R^* = \tau\frac{(\mu+\gamma)(\mu+\tau)(\mu+\theta+\alpha)-\gamma\tau(\mu+\theta+\alpha)-\beta\lambda(\mu+\tau)}{\beta(\mu+\tau)\left[\sigma\tau-(\mu+\sigma)(\mu+\theta+\alpha)\right]} \quad , \quad V^* = \gamma\frac{(\mu+\theta+\alpha)}{\beta(\mu+\tau)}$$

(6)

If the vaccination is less than a certain critical value, the disease persist.

**Theorem 2**

If $\gamma < \frac{\beta\lambda}{(\mu+\theta+\alpha)} - \mu$, then the endemic equilibrium point become unstable. The disease will persist.

**Proof**

From the Jacobian matrix J=

$$J = \begin{pmatrix} -(\mu+\gamma) & -\frac{\beta\lambda}{\mu+\gamma} & \sigma & \tau \\ 0 & \beta\frac{\lambda}{\mu+\gamma}-(\mu+\theta+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & 0 & -(\mu+\tau) \end{pmatrix}$$

The determinant is greater than zero

$$\frac{\beta\lambda}{(\mu+\gamma)}-(\mu+\theta+\alpha) > 0 \qquad (7)$$

Re-arranging (7) yields

$$\frac{\beta\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}-1 > 0 \qquad (8)$$

But from (4), $R_0 = \dfrac{\beta\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}$

Therefore, $R_0 > 1$. (9)

The endemic equilibrium will only occur if $R_0 > 1$. This means that the disease become unstable and the rest point is lost. The vaccinated animals loose their immunity and become susceptible.

**Lemma 2**

If $\gamma > \dfrac{\beta\lambda}{\mu+\theta+\alpha} - \mu$, the endemic equilibrium point becomes stable. Therefore, the disease persists.

The table below shows sensitivity analysis on how each parameter contribute to the basic reproductive ratio $R_0$ of the model. Sensitivity analysis is given by the relation:

$$S_A^{R_0} = \frac{\partial R_0}{\partial A} \times \frac{A}{R_0} \ .$$

Where A is any parameter used in the model.

| **Parameter** | **Contribution** | **Baseline values** | **References** |
|---|---|---|---|
| $\lambda$ | Positive | 200 | [4] |
| $\beta$ | Positive | 0.0001 | [1,7] |
| $\mu$ | Negative | 0.001 | Estimate |
| $\gamma$ | Negative | 0.10 | [7] |
| $\sigma$ | Negative | 0.02 | Estimate |
| $\tau$ | Negative | 0.003 | [3] |

| | | | |
|---|---|---|---|
| $\theta$ | Negative | 0.15 | [1] |
| $\alpha$ | Negative | 0.01 | [7] |

## 3.0 Results and Discussion

In this study, we modeled vaccination compartment in the transmission dynamics of anthrax in animal population. The outcome of stability analysis of the endemic equilibrium state shows that it is possible to effectively control anthrax outbreak in animal population.

Taking the initial conditions for endemic equilibrium $\varepsilon^*( \quad S^* = 2000, I^* = 100, R^* = 300, V^* = 500)$ and time t= 10 years and considering parameters baseline values from other literature as indicated above, the reproductive ratio $R_O$ =1.2299. Increasing the rate of vaccination $\gamma$ , the reproductive ratio $R_O$ decreases. Therefore, animals will not die as a result of anthrax infection. When $\gamma$ is increased by 24.5%,the reproductive ratio $R_O$ decrease by 19.50%.In this case, reproductive ratio become 0.9900 which is less than unit. Hence, the disease free equilibrium.

The outcome of the model shows that vaccination is a good control strategy against anthrax outbreak in animal population. However, vaccination may not completely guarantee protection of the animals against anthrax but it is possible that the vaccinated animals with time may lose immunity and may contract anthrax disease again. Therefore, there is need to keep vaccinating animals periodically against anthrax to keep anthrax prevalence as low as possible or completely eradicated.

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