Advantages of the Mathematical Structure of a Dirac Fermion

Abstract

The mathematical structure of quantum field theories of first order and of second order partial differential equations is analyzed. Relativistic properties of the Lagrangian density and the dimension of its elements are examined. The analysis is restricted to elementary massive particles that are elements of the Standard Model of particle physics. In the case of the first order Dirac equation, the dimensionless 4-vector γ^μ and the partial 4-derivative ∂_μ whose dimension is $[L^{-1}]$, are elements of the mathematical structure of the theory. On the other hand, the mathematical structure of second order quantum equations has no dimensionless 4-vector which is analogous to γ^μ of the linear equation. It is proved that this deficiency is the root of inherent theoretical inconsistencies of second order quantum equations. Problems of the Klein-Gordon particle, the electroweak theory of the $W^\pm$, Z particles and the Higgs boson theory are discussed.

Keywords: Quantum Field Theory, Lagrangian Density, First Order and Second Order Quantum Equations, Electromagnetic Interaction, Consistency Tests

1 Introduction

The variational principle plays a primary role in the present structure of physical theories. In the case of classical mechanics of massive particles, this principle uses a Lagrangian, whereas quantum theories are derived from a Lagrangian density. This approach is adopted by contemporary textbooks. For example: the variational principle is "the foundation on which virtually all modern theories are predicated" (see [1], p. 353). The general form of a Lagrangian density of a quantum theory is

$$\mathcal{L}(\psi(x), \psi(x)_{,\mu}), \tag{1}$$

where the standard notation is used. Another textbook supports this approach and states that "All field theories used in current theories of elementary particles have Lagrangians of this form" (see [2], p. 300). The Lagrangian density (1) is regarded as the main expression of each quantum theory, and the quantum equations of motion are partial differential equations that are the Euler-Lagrange equations which are derived from the variational principle

$$\frac{\partial \mathcal{L}}{\partial \psi} - \frac{\partial}{\partial x^\mu} \frac{\partial \mathcal{L}}{\partial(\partial \psi / \partial x^\mu)} = 0 \tag{2}$$

(see [2], p. 300, [3], p. 17). This principle is used below in an examination of the mathematical structure of quantum field theory (QFT) of several kinds of elementary massive particles.

Various arguments support this approach. Special relativity is a well-established theory and any QFT should be consistent with it. This requirement is satisfied if the Lagrangian density is a Lorentz scalar. For example, it is stated that "the point of the Lagrangian formalism is that it makes it easy to satisfy Lorentz invariance and other symmetries: a classical theory with a Lorentz-invariant Lagrangian density will when canonically quantized lead to a Lorentz-invariant quantum theory" (see [2], p. 292). Furthermore, physical processes abide by conservation laws, like those of energy, momentum and angular momentum. The Noether theorem proves that the Lagrangian density (1), which does not explicitly depend on space-time coordinates, yields equations of motion that satisfy conservation of energy, momentum and angular momentum (see e.g. [3], pp. 17-22)

The main objective of this work is to use the above mentioned framework for an examination of the mathematical structure of quantum theories of first order partial differential equations and of quantum theories of second order partial differential equations. The analysis proves the consistence of the first order Dirac theory. In contrast, unsettled problems exist with second order quantum theories of elementary massive particles, like those of the Klein-Gordon (KG), $W^{\pm}$, Z and the Higgs bosons, which are described by second order partial differential equations (see [4], pp. 16, 17, 701, 715). Quotations from the present mainstream literature support this conclusion.

This work uses units where $\hbar = c = 1$. It follows that the action is dimensionless and the dimension of a Lagrangian density $\mathcal{L}$ is $[L^{-4}]$. Formulas take the standard form of a relativistic covariant expression. The metric is diagonal and its entries are (1,-1,-1,-1). The second section present several constraints that apply to an acceptable quantum theory. The third section shows that the first order Dirac equation is consistent with these constraints. The fourth section presents inconsistencies of second order quantum theories of massive particles. The fifth section contains a further discussion of these issues. The last section summarized this work.

2 Constraints on Quantum Theories

Constraints on the structure of a physical theory are useful elements because they prevent a construction of a theory that is inconsistent with well-established physical laws. The necessity to abide by relativistic covariance and conservation laws is already mentioned above in the introduction section. Other constraints on QFT of massive particles are listed below.

C.1 $\hbar$ has the dimension of action. Hence, in units where $\hbar = c = 1$ the action is dimensionless, and the dimension of the Lagrangian density $\mathcal{L}$ of (1) is $[L^{-4}]$. This property determines the dimension of the quantum functions ψ of $\mathcal{L}$. Obviously, expressions that depend on quantum functions must satisfy dimensional balance. This issue is used below in several cases. Furthermore, the discussion presents examples that indicate that this self-evident attribute of the quantum functions of $\mathcal{L}$ is apparently not well known.

C.2 The nonrelativistic limit of QFT corresponds to ordinary quantum mechanics. Here is a quotation that clearly states this issue. "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear and condensed matter physics" (see [2], p. 49). Below, this requirement is called the Weinberg correspondence principle.

C.3 Many textbooks explain the correspondence between the classical limit of quantum mechanics and classical physics. For example: "Classical mechanics must therefore be a limiting case of quantum mechanics" (see [5], p. 84; see also [6], p. 15). This issue is called the Bohr correspondence principle. Hence, the Weinberg correspondence principle together with the Bohr correspondence principle mean that an appropriate limit of QFT corresponds to classical physic.

C.4 An elementary classical massive particle is pointlike (see [7], pp. 46, 47). Hence, particle's position is well-defined in classical physics. The uncertainty principle says that the position of a quantum particle is approximately described by an expression for its density. The correspondence principles C.2 and C.3 prove that a theory of an elementary massive quantum particle must provide a consistent expression for density, namely for the j^0 component of a conserved 4-current

j^μ.

C.5 The interaction term of Maxwellian electrodynamics is a contraction of a conserved 4-current with the electromagnetic 4-potential

$$\mathcal{L}_{int} = ej^\mu A_\mu \tag{3}$$

(see [7], p. 75). This is another reason for the need of a consistent expression for a conserved 4-current of an elementary charged particle.

C.6 The Noether theorem provides an expression for a conserved 4-current of a quantum particle. Assume that the particle's Lagrangian density is invariant under a global phase transformation of the quantum function

$$\psi(x) \to e^{i\alpha}\psi(x), \tag{4}$$

where α is a real variable (see [8], p. 314). An application of this transformation yields

$$0 = i\alpha\left[\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right)\right]\psi + i\alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right) \tag{5}$$

The expression inside the square brackets vanishes due to the Euler-Lagrange equation (2). Furthermore, the variation parameter $\alpha \neq 0$ means that the expression inside the last brackets represent a conserved 4-current

$$j^\mu_{,\mu} = 0, \tag{6}$$

where

$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi. \tag{7}$$

Here j^0 is the required density. It is interesting to note that a nonvanishing contribution to a Noether 4-current is obtained from terms of the Lagrangian density that contain a derivative $\partial_\mu\psi$ of the quantum function. This important property of the Noether 4-current (7) is mentioned below in several cases.

C.7 Quantum states can be organized as elements of a Hilbert space (see [2], pp. 49, 50). This space requires a well defined inner product of any pair of its elements.

C.8 Observables are represented by Hermitian operators that apply to elements of a Hilbert space (see [2], p. 50). In particular, let ψ be a normalized eigenfunction of an operator A whose eigenvalue is α, then

$$A\psi = \alpha\psi \ \text{ and } \ (\psi, A\psi) = \alpha, \tag{8}$$

where the second expression is the inner product of the Hilbert space. The primary objective of a physical theory is to provide a good description of experimental data. Hence, a quantum theory must have a well-defined form of relevant observables. In particular, the Hamiltonian is a vital element of quantum mechanics. Therefore, the Weinberg correspondence principle means that QFT must provide a consistent expression for the Hamiltonian.

Below, each of these requirements is denoted by C.n, where n is the figure of the respective requirement.

Here is a simple example that explains the vital need for a consistent expression for some of the above mentioned quantities. Consider the leptonic decay of the Z particle [9]

$$\mu^- \leftarrow Z \rightarrow \mu^+. \tag{9}$$

Experimental devices measure the $(t, \mathbf{x})$ values of the outgoing μ^-, μ^+ leptons and their energy-momentum. These data determine the trajectory of each of the outgoing particles. If the two trajectories have a common space-time very small region that belongs to the common region of the primary colliding beams, and if the invariant energy of the two particles agrees with that of the Z boson then the event is recognized as a Z decay. The decay (9) is a particle creation and destruction process, which belongs to the QFT domain of validity. It follows that an acceptable QFT theory must provide appropriate expression for density and for energy-momentum of particles. This example explains the relevance of requirements C.4, C.5 and C.8 to the real world.

3 The Dirac Equation

The Lagrangian density of a free Dirac particle is

$$\mathcal{L}_D = \bar{\psi}(\gamma^\mu i\partial_\mu - m)\psi, \tag{10}$$

where $\bar{\psi} \equiv \psi^\dagger\gamma^0$ (see [3], p. 54, [4], p. 78). As required, the Lagrangian density (10) is a Lorentz scalar. The $[L^{-4}]$ dimension of a Lagrangian density and the linearity of (10) prove that the dimension of a Dirac function ψ is $[L^{-3/2}]$. The Lorentz invariance of the first term of (10) is obtained from a contraction of *two* different 4-vectors: γ^μ and ∂_μ.

An important feature of (10) is that it is *not* a symmetric expression with respect to $\bar{\psi}$, ψ. Indeed, (10) contains a derivative of ψ but it is free of a derivative of $\bar{\psi}$. This issue plays an important role in the structure of the Dirac 4-current.

The Lagrangian density (10) is invariant under the global phase transformation

$$\psi(x) \rightarrow e^{i\alpha}\psi(x), \tag{11}$$

where α is a real constant (see [8], p. 314). In this case, the Noether theorem C.6 yields an expression for a conserved 4-current whose form is

$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi = \bar{\psi}\gamma^\mu\psi, \tag{12}$$

(see e.g. [3], p. 56). The density of the Dirac particle is

$$j^0 = \psi^\dagger\psi, \tag{13}$$

where the relation $\bar{\psi}\gamma^0 = \psi^\dagger$ is used. It is interesting to note that *while the Dirac Lagrangian density* (10) *is not symmetric with respect to* $\bar{\psi}$, ψ, *its associated 4-current* (12), *is symmetric with respect to these functions!*

The symmetric 4-current (12) plays an important role in the structure of the Dirac theory. Consider for example the Dirac Lagrangian density (10). The Hamiltonian density that is obtained from (10) is

$$\mathcal{H}_D = \psi^\dagger[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m]\psi, \tag{14}$$

where $\boldsymbol{\alpha}$, β denote the four Dirac matrices (see e.g. [3], p. 55). This form shows that the quantity inside the brackets of the (14) is the *operator* form of the Dirac Hamiltonian, which stands between the functions $\psi^\dagger$, ψ. Since $\psi^\dagger\psi$ is the Dirac density, one finds the well-known form of the *Hamiltonian operator* of a free Dirac particle

$$H_D = -i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m \tag{15}$$

(see [4], p. 52). Evidently, the Dirac Hamiltonian is a Hermitian operator. The $\psi^\dagger\psi$ Dirac density means that the 3-dimensional integration of (14) takes the required inner product of the Hilbert space (8)

$$(\psi, H_D\psi) \equiv \int d^3x\, \psi^\dagger[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m]\psi. \tag{16}$$

This expression for the Dirac Hamiltonian satisfies requirement C.8.

The fact that the Dirac 4-current (12) is independent of derivatives of ψ is a crucial property, which is used in its electromagnetic interaction

$$\mathcal{L}_{int} = -ej^\mu A_\mu = -e\bar{\psi}\gamma^\mu\psi A_\mu \tag{17}$$

(see [2], p. 349, [3], p. 84). This term is free of derivatives of the fields, which means that the introduction of electromagnetic interaction does not change the Noether 4-current (12). Hence, the Dirac theory is consistent with requirements C.4, C.5. Moreover, the fact that density does not change means that the interaction term (17) does not affect the inner product of the Hilbert space, and requirement C.7 holds.

4 Inherent Problems of Second Order Quantum Equations

A second order quantum field theory of a massive particle is derived from a Lagrangian density whose general form is

$$\mathcal{L} = \phi^\dagger_{,\mu}\phi_{,\nu}g^{\mu\nu} - m^2\phi^\dagger\phi + OT, \tag{18}$$

where OT denotes other terms. The first term of (18) is a Lorentz-contraction of two 4-gradients of the field functions $\phi^\dagger$, ϕ. In some cases a Lorentz-contraction of two 4-curls replaces the first term of (18). This term is bilinear in derivatives of $\phi^\dagger$, ϕ. Hence, the second term of the Euler-Lagrange equation (2) yields a second order partial differential equation. The first and the second terms of (18) are the KG Lagrangian density (see e.g. [2], p. 21, [10], p. 191).

Textbooks show that the first term of (18) yields a Noether 4-current that is *antisymmetric* with respect to $\phi^\dagger$, ϕ

$$j_\mu = i(\phi^\dagger\phi_{,\mu} - \phi^\dagger_{,\mu}\phi) \tag{19}$$

(see e.g. [2], p. 27, [3], p. 40, [10], p. 193). In particular, the expression for density, j^0 is *antisymmetric* with respect to $\phi^\dagger$, ϕ. In contrast, the Hamiltonian density is the T_{00} component of the energy-momentum tensor. Hence, the Hamiltonian density that is derived from (18) is *symmetric* with respect to $\phi^\dagger$, ϕ. For example, the KG Hamiltonian density is

$$\mathcal{H} = \phi^\dagger_{,0}\phi_{,0} + \sum_{i=1}^{3}\phi^\dagger_{,i}\phi_{,i} + m^2\phi^\dagger\phi \tag{20}$$

(see e.g. [2], p. 22, [3], p. 38, [10], p. 192).

The opposite $\phi^\dagger$, ϕ symmetry of the density (19) and of the Hamiltonian density (20) prove that in the case of a second order quantum theory one cannot extract the *Hamiltonian operator* from the Hamiltonian density. This shortcoming differs from the corresponding feature of the first order Dirac theory which provides an explicit form of the Hamiltonian operator (see (14), (15)). Therefore, second order quantum theories of a massive particle are inconsistent with the Weinberg correspondence principle, because the *Hamiltonian operator* is a crucial element of quantum mechanics.

Another discrepancy of a second order quantum theory stems from the density j^0 of (19). Here density depends on *time-derivative* of the field functions $\phi^\dagger$, ϕ. Therefore, the Heisenberg picture cannot be used for this theory, because field functions of this picture are time-independent. Hence, in the case of second order quantum equation, one cannot be sure of a physical property whose validity relies on the Heisenberg picture (see e.g. [2], pp. 109, 288, 297, 298, 425).

A special problem exists in the case of an electrically charged particle that belongs to a second order quantum theory, like the charged KG particle and the electroweak $W^\pm$ bosons. Here, neither of the following alternatives describes properly electromagnetic interaction.

Q.1 Consider an application of the transformation

$$\partial_\mu \rightarrow \partial_\mu - eA_\mu \tag{21}$$

to a Lagrangian density (see e.g. [2], p. 9, [10], p. 198). This transformation is called the *minimal interaction*. The first term of (18) proves that in second order theories the transformation (21) yields a Lagrangian density that depends *quadratically* on the 4-potential A_μ. Hence, Maxwellian electrodynamics is violated [7].

Q.2 The original form of the Noether 4-current (19) is used in an expression of the electromagnetic interaction

$$\mathcal{L}_{int} = [i(\phi^\dagger \phi_{,\mu} - \phi^\dagger_{,\mu}\phi)]A^\mu. \tag{22}$$

This term depends explicitly on derivatives. Hence, it destroys the Noether expression for the 4-current (19) upon which it depends.

Q.3 The general structure of a second order Lagrangian density (18) and its Noether 4-current (19) demonstrate an intrinsic difference between the Dirac linear quantum theory and theories that have a second order equation: As stated above, in the case of a Dirac theory, the Lagrangian density (10) is not symmetric with respect to $\bar{\psi}$, ψ, whereas the corresponding 4-current (12) is symmetric with respect to these functions. In contrast, in a second order quantum theory the Lagrangian density (18) is symmetric with respect to $\phi^\dagger$, ϕ, but the corresponding Noether 4-current (19) is *antisymmetric* with respect to these functions. It is shown above that this quite unfavorable property of a second order quantum theory disables a construction of a Hamiltonian operator, which is required by the Weinberg correspondence principle.

The literature provides strong evidence that indicates the correctness of the foregoing result, which proves that no consistent expression can describe the electromagnetic interaction of an electrically charged elementary quantum particle that satisfies a second order quantum equation. For this purpose, let us compare the status of electromagnetic interactions of the Dirac first order quantum equation with that of second order quantum equations. In the case of the first order quantum theory, electromagnetic interaction is correctly described in the *original* Dirac paper (see eq. (14) in [11]). Furthermore, an explicit expression of a conserved 4-current of a Dirac particle has been found about one month later [12]. By contrast, many decades have already elapsed since the rise of the electroweak theory but very large research centers, like CERN and Fermilab, still use *effective expressions* for the electroweak description of the $W^\pm$ electromagnetic interactions [13,14]. Here the *effective expression* violates Maxwellian electrodynamics because its interaction term contains derivatives, and it is not based on a consistent 4-current.

Here are two quotations from textbooks that provide another support for the claim about the discrepancy of a second order quantum theory of a charged particle with respect to Maxwellian electrodynamics. "... electrodynamics of spinless particles is more complicated" (see [2], p. 349). Another statement describes problems of electromagnetic interactions of a charged KG particle: "Indeed, they appear with a vengeance, since the coupling prescription (15.1) introduces interaction terms containing derivatives" (see [3], p. 87). (Note that (15.1) of this textbook is the above mentioned minimal interaction (21).)

5 Discussion

It is pointed out above that the first order Dirac theory can use the dimensionless 4-vector γ^μ and the partial 4-derivative ∂_μ whose dimension is $[L^{-1}]$ as elements of the theory, whereas second order quantum theories have no analog for the γ^μ 4-vector of the Dirac theory. An interaction of a quantum particle with an external second-rank antisymmetric tensor provides an example that illustrates this drawback of second order theories.

The idea that the electron may also interact directly with external electromagnetic field has been suggested a long time ago (see [15], p. 223, [2], pp. 14, 517, 520). The corresponding interaction, which is called the Pauli term, takes the form

$$\mathcal{L}' = \mathrm{d}\bar{\psi}\sigma_{\mu\nu}\mathcal{F}^{\mu\nu}\psi, \tag{23}$$

where

$$\sigma_{\mu\nu} \equiv \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) \tag{24}$$

(see [16], p. 21), and the coefficient d has the dimension of length. The interaction (23) alters the Dirac expression for the electron's dipole moment (see [15], p. 223, [2], p. 14). As a matter of fact, the ordinary Dirac interaction (17), which contains no term like (23), yields a very good prediction for the electron's magnetic dipole moment. Hence, the Pauli term (23) has been removed from the standard expression for the electron's *electromagnetic interaction*.

The Pauli term has recently been rediscovered, and it can be shown that it describes weak interactions, where parity violation is *proved* [17–19]. Here the transition from the Lagrangian density to the Hamiltonian density adds a γ^0 factor, and the product $\gamma^0\sigma_{\mu\nu}$ of (23) splits into a sum of a vector and an axial vector. The Pauli term (23) shows the flexibility of the first order Dirac theory, where the dimensionless γ^μ 4-vector enables to write down a consistent *derivative-free* covariant expression for the interaction of a Dirac particle with a second rank antisymmetric field tensor that takes the form of $F^{\mu\nu}$.

Problems arise if an analogous attempt is made with the $W^\pm$, which the electroweak theory regards as an elementary charged particles belonging to the second order category of quantum theories. A term that represents the Standard Model $W^\pm$ interaction with an external second rank antisymmetric tensor $V^{\nu\eta}$ is

$$\mathcal{L}_{WWV} = iW_\mu^\dagger W_\nu V^{\mu\nu} \tag{25}$$

(put $k_v = 1$ in eq. (3) of [14]. See also [13], [20], [21]).

Dimensional considerations totally reject this expression. Indeed, as shown above, the dimension of the electroweak quantum function $W^\pm$ is $[L^{-1}]$. It means that the dimension of the product $W_\mu^\dagger W_\nu$ of (25) is $[L^{-2}]$. This value disagrees with the $[L^{-3}]$ dimension of the electric charge density. Therefore, the electroweak interaction term (25) strongly violates Maxwellian electrodynamics, where the interaction is proportional to the strength of the electric charge.

It is well known that dimensional balance is a very strong requirements that every physical expression must abide with. Requirement C.1 states that the Lagrangian density of the variational principle determines the dimension of the field functions ψ

of (1). The term (25), which violates dimensional balance of charge density, is used in [13], [14], [20], [21], and the total number of the authors of these publications is a number of four decimal digits. This evidence indicates that the dimension attribute of a quantum function is still not very well known.

Some points of this work explain why the electroweak theory suffers *unsettled* contradictions. The following items demonstrate one issue.

EW.1 The electroweak theory is based on a Lagrangian density and the dimension of each of its terms is $[L^{-4}]$.

EW.2 The second order of the differential equations of the electroweak theory proves that the dimension of each of its quantum functions is $[L^{-1}]$.

EW.3 The electroweak theory regards the $W^{\pm}$ as elementary charged particles. Maxwellian electrodynamics is based on a conserved 4-current. The 4-current's dimension is $[L^{-3}]$, and it satisfies the continuity equation (6).

EW.4 The dimension of the product of the electroweak functions $W^{\dagger}W$ is $[L^{-2}]$. Therefore, dimensional balance requires that *any 4-current of the $W^{\pm}$ must depend on a derivative with respect to the space-time coordinates.*

EW.5 Items Q.1 and Q.2 of section 4 prove that a derivative destroys the compatibility of the $W^{\pm}$ electromagnetic interaction.

EW.6 The straightforward observation of real facts which is presented near the end of the previous section relies on commonsense. These facts provide a very strong support for the inability to find a consistent expression for the 4-current of the electroweak $W^{\pm}$ particles: After about half a century, the literature still does not show a consistent 4-current of the electroweak $W^{\pm}$; CERN and Fermilab use an electromagnetic interaction term of the $W^{\pm}$ whose 4-current does not satisfy the continuity equation (6).

6 Concluding Remarks

This work shows the consistency of the Dirac theory of a massive quantum particle, that is described by first order partial differential equation with respect to the four space-time coordinates. In particular, the 4-current (12) and the associated density (13) are consistently described. These variables enable the extraction of the Dirac Hamiltonian operator from the Hamiltonian density. This objective is required for the Weinberg correspondence between QFT and quantum mechanics. Moreover, it is well-known that the Dirac 4-current enables a consistent description of electromagnetic interaction (17).

By contrast, inherent problems hold for massive quantum particles, like those of the KG, $W^{\pm}$, Z and the Higgs bosons, which are described by second order partial differential equations (see [4], pp. 16, 17, 701, 715). In particular, no consistent expression for density holds, and electromagnetic interactions of charged particles are described by non-Maxwellian phenomenological expressions. Furthermore, a consistent *Hamiltonian operator* cannot be extracted from the Hamiltonian density, and a Hilbert space cannot be constructed. Hence, the Weinberg correspondence principle fails.

The flexibility of the Dirac theory, which has two different 4-vectors, γ_μ and ∂_μ, is shown as a useful theoretical element. These 4-vectors enable a construction of a consistent Lagrangian density which is a Lorentz scalar whose dimension is $[L^{-4}]$. Second order quantum theories have no analog for the Dirac γ_μ 4-vector, and they have no consistent expressions for the Lagrangian density. In particular, no consistent 4-current exists and it is proved above that the fundamental structure of Maxwellian electrodynamics fails. Moreover, no *Hamiltonian operator* exists, and the time-independent Heisenberg picture cannot be used. It is also shown that an indirect support for these conclusions can be found in mainstream scientific literature.

The present structure of the Standard Model (SM) of particle physics is based on the above mentioned theories of the $W^{\pm}$, Z and the Higgs bosons. This work proves that the SM suffers fundamental problems.

It is interesting to mention that these results agree with Dirac's lifelong objection to second order equation of a massive quantum particle (see [2], p. 14, [22], p. 3).

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