

Modelling and allocation of vegetable crops using Mathematical Programming

Abstract

Mathematical programming techniques are commonly used by decision makers and achieving efficiency in agricultural production planning. Due to increasing demands of growing population of world, one needs to utilize the limited available resources in the most efficient and economic way. In this paper, the fractional programming problem is formulated and is used to determine the optimal cropping pattern of vegetable crops in such a way that the total profit is maximized. The solution of the formulated Fuzzy programming problem is obtained using LINGO.

Keywords: optimal solution, optimal land allocation, Fractional goal programming, Multiobjective linear programming Problem.

Introduction:

In agricultural field experiments, crop planning is usually carried out to determine which type of crops should be cultivated and the area required for planting the crop. This planning issue is usually solved by using Mathematical programming techniques. Linear programming is one of the oldest techniques of Mathematical programming used for decision making studies. The most ordinary kind of mathematical programming is Fractional programming Romero and Rehman (1989) with ratio of objectives. Due to increasing demands of growing population of world the manufacture may have to invest a little more than the initial proposed budget in the interest of his production process. In this situation fuzzy set theory can be used to formulate the model with the help of membership functions. Most of the applications in agricultural planning correspond to the problem of determining an optimum-cropping pattern with multiple goals. Goal Programming techniques have been successfully used for these purposes Romero (1991). Multiobjective linear plus linear fractional programming problem solutions are found in Hirche (1984), Chadha (1993), Jain and Lachhwani (2008), Schaible (1977), Dangwal *et.al* (2012), Lone *et.al* (2015) etc. The first mathematical formulation of fuzziness was pioneered by Zadeh (1965). Orlovsky (1980) made a numerous attempts to explore the ability of fuzzy set theory to become a useful tool for adequate mathematical analysis of real world problems. Fuzzy methods have been developed in virtually all branches of decision making problems can be found in Tamiz (1996), Zimmermann (1991) , Kumari *.et.al*(2014) and Ross (1995). Goal programming approach in fuzzy environment has been first introduced by Narashimann (1980).Fuzzy goal programming has been discussed by several authors (see Pal *et al.* (2003) and Parra *et al.* (2001) , Lone *et.al.*(2016) etc.).

In this paper we have demonstrated that how a farmer who has limited resources such as availability of labor work time, water and land on which he/she wanted to grow three vegetable crops, Bringal, Tomato and ladies finger. The farmer's objective is to determine the optimal cropping pattern so that the total profit will be maximized

Linear Fractional Programming

A problem in which the objective function is the ratio of two linear functions and constraints are linear. Such problems are called linear fractional programming problems and can be stated precisely as follows:

$$\begin{aligned} & \text{Optimize } Z = \frac{p'x + \alpha}{q'x + \beta} \\ & \text{subject to} \\ & \quad Ax = b \\ & \quad x \geq 0 \end{aligned}$$

where p and q are n vectors, b is an m vector. A is $m \times n$ matrix. α and β are scalars. If an optimal solution for a linear fractional problem exists, then an extreme point optimum exists.

Mathematical Formulation of General Multi-objective Programming Problem

The general multi-objective programming problem with n decision variables, m constraints and p objective is:

$$\begin{aligned} & \text{Optimize } Z = Z(X_1, X_2, \dots, X_n) \\ & \quad = [Z_1(X_1, X_2, \dots, X_n), \\ & \quad \quad Z_2(X_1, X_2, \dots, X_n), \\ & \quad \quad \dots, Z_p(X_1, X_2, \dots, X_n)] \end{aligned} \quad \left. \vphantom{\begin{aligned} & \text{Optimize } Z = Z(X_1, X_2, \dots, X_n) \\ & \quad = [Z_1(X_1, X_2, \dots, X_n), \\ & \quad \quad Z_2(X_1, X_2, \dots, X_n), \\ & \quad \quad \dots, Z_p(X_1, X_2, \dots, X_n)] \end{aligned}} \right\} \quad (D)$$

subject to

$$\begin{aligned} & g_i(X_1, X_2, \dots, X_n) \leq 0 \\ & \text{and } X_j \geq 0, \quad (i=1, 2, \dots, m, j=1, 2, \dots, n) \end{aligned}$$

where, $Z(X_1, X_2, \dots, X_n)$ is the multi-objective function with $Z_1(X_1, X_2, \dots, X_n)$, $Z_2(X_1, X_2, \dots, X_n)$, \dots , $Z_p(X_1, X_2, \dots, X_n)$ are p individual objective functions.

For multi-objective linear programming problem (MOLPP) the proposed approach can be outlined as given below:

Step 1: solve problem with each single objective. Here $P=3$ and Find the minimum value of $MaxZ1, MaxZ2, \text{ and } MaxZ3$, suppose $MaxZ2$, has minimum optimal value.

Step 2: Divide each objective individually say by $MaxZ2$, .

Step 3: we get fractional programming $\xi_1(z1(x))$, and $\xi_2(z2(x))$

Step 4: Define the membership function for P^{th} objective.

If $Z_p(x) \leq g_p$ then

$$\mu_p(x) = \begin{cases} 1 & \text{if } Z_p(x) \leq g_p \\ \frac{u_p - Z_p(x)}{u_p - g_p} & \text{if } g_p \leq Z_p(x) \leq u_p \\ 0 & \text{if } Z_p(x) \geq u_p \end{cases}$$

If $Z_p(x) \geq g_p$ then

$$\mu_l(x) = \begin{cases} 1 & \text{if } Z_p(x) \geq g_p \\ \frac{Z_p(x) - l_p}{g_p - l_p} & \text{if } l_p \leq Z_p(x) \leq g_p \\ 0 & \text{if } Z_p(x) \leq l_p \end{cases}$$

where g_p is the aspiration level of the p^{th} objective $Z_p(x)$ and u_p and l_p ($p= 1, 2, \dots, m$) are the upper tolerance limit and lower tolerance limit, respectively, for the p^{th} fuzzy goal. Zimmermann (1983) presented a fuzzy approach to multi-objective linear programming problems. Now, we formulate the fuzzy programming model of problem (D) by transforming the objective functions into fuzzy goals by assigning aspiration level to each of them using Zimmermann (1978) Max-min approach.

Step 5: Now, transform non linear membership functions $\mu_p(x)$ into an equivalent linear membership functions at individual best solution point by using first order Taylor's series as follows:

$$\mu_p(x) = \mu_p(x_p^*) + [(x_1 - x_{p1}^*) \frac{d\mu_p(x_p^*)}{dx_1} + (x_2 - x_{p2}^*) \frac{d\mu_p(x_{pt}^*)}{dn_2} + \dots + (x_L - x_{pL}^*) \frac{d\mu_p(x_p^*)}{dx_L}]$$

where x_i^* is the individual best solution.

Step 6: Solve the fuzzy goal problem using LINGO.

Numerical Illustration

A farmer has 8 acres farm on which he/she grow three vegetable crops, Bringal, Tomato and ladies finger. As per his/her past his expense, the total available labor work time and total availability of water are 200 (000hrs hours) and 30(acre-inches) respectively. The information related to total profit in lakhs obtained from these three crops for one acre of land is given in the tabular form below. Now, the farmer's objective is to determine the optimal cropping pattern so that the total profit will be maximized. This example is a variant of [Kumari et al. \(2014\)](#).

Vegetable crops	Profit (plot1)	Profit (plot2)	Profit (plot3)	Labor requirement (00hrs)	Water requirement/acre-inches
Bringal	1.20	2.30	0.48	1.40	20.4
Tomato	0.35	0.45	1.15	1.30	17.5
Ladies finger	1.60	0.30	0.90	1.80	24.5 (24.4)

Please put X_1, X_2, X_3 in capital

Let x_1 (X_1) be the area required in acres for Bringal crop.

Let x_2 (X_2) be the area required in acres for Tomato crop and

Let x_3 (X_3) be the area required in acres for ladies finger crop.

Therefore, the multi objective problem can be formulated as

$$\text{Max}K_1 = 1.20X_1 + 2.30X_2 + 0.48X_3$$

$$\text{Max}K_2 = 0.35X_1 + 0.45X_2 + 1.15X_3$$

$$\text{Max}K_3 = 1.60X_1 + 0.30X_2 + 0.90X_3$$

Subject to

$$X_1 + X_2 + X_3 \leq 8$$

$$1.40X_1 + 1.30X_2 + 1.80X_3 \leq 200$$

$$20.4X_1 + 17.5X_2 + 24.4X_3 \leq 30$$

Using step 1, we get

$$\text{Max}K_1 = 3.94, (0, 1.71, 0)$$

$$\text{Max}K_2 = 1.41, (0, 0, 1.23)$$

$$\text{Max}K_3 = 2.35, (1.47, 0, 0)$$

Using step (2 and 3), we have

$$\text{Max}\zeta_1$$

$$\text{Max}\zeta_2$$

Subject to

$$X_1 + X_2 + X_3 \leq 8$$

$$1.40X_1 + 1.30X_2 + 1.80X_3 \leq 200$$

$$20.4X_1 + 17.5X_2 + 24.4X_3 \leq 30$$

After solving this we get

$$\text{Max}\zeta_1 = 5.11, (0, 0.94, 0)$$

$$\text{Max}\zeta_2 = 4.57, (0.32, 0, 0)$$

It is observed that $\text{Max}\zeta_1 \geq 0$, and $\text{Max}\zeta_2 \geq 0$

Using step 4 and 5, we have

$$\zeta_1(x) = -1.39X_1 - 12X_3 + 5.11$$

$$\zeta_2(x) = -0.16X_1 - 0.154X_3 + 4.57$$

Thus the fractional programming problem is now transferred in Linear programming. The fuzzy goal programming is as follows with their possible aspiration levels as given below:

Modify 05.11 to 5.11

$$\zeta_1(x) = -1.39X_1 - 12X_3 + 5.11 \leq 05.11$$

$$\zeta_2(x) = -0.16X_1 - 0.154X_3 + 4.57 \leq 4.57$$

Subject to

$$X_1 + X_2 + X_3 \leq 8$$

$$1.40X_1 + 1.30X_2 + 1.80X_3 \leq 200$$

$$20.4X_1 + 17.5X_2 + 24.4X_3 \leq 30$$

For author: Please give justifications for selecting the tolerance limits to be 6,5. What happen if we choose another tolerance limits.

Let (6, 5) be the tolerance limits for two goals respectively. The membership function can be defined for both of the two goals as

For author: Note that 0.83 is 0.89 = (6-5.11)

$$\mu_1(x) = \begin{cases} 1 & \text{if } \zeta_1(x) \leq 5.11 \\ \frac{6 - \zeta_1(x)}{0.83} & \text{if } 5.11 \leq \zeta_1(x) \leq 6 \\ 0 & \text{if } \zeta_1(x) \geq 6 \end{cases}$$

For author:

For lower limet the Eq. is

$$\mu_2(x) = \begin{cases} 1 & \text{if } \zeta_2(x) \leq 4.57 \\ \frac{\zeta_2(x) - 5}{0.43} & \text{if } 4.57 \leq \zeta_2(x) \leq 5 \\ 0 & \text{if } \zeta_2(x) \geq 5 \end{cases}$$

Not

$$\mu_2(x) = \begin{cases} 1 & \text{if } \zeta_2(x) \leq 4.57 \\ \frac{5 - \zeta_2(x)}{0.43} & \text{if } 4.57 \leq \zeta_2(x) \leq 5 \\ 0 & \text{if } \zeta_2(x) \geq 5 \end{cases}$$

So what is the new solution.

Now, using step 6, Fuzzy goal programming can be formulated as

$$\text{Max } G = \mu_1 + \mu_2$$

Subject to

$$X_1 + X_2 + X_3 \leq 8$$

$$1.40X_1 + 1.30X_2 + 1.80X_3 \leq 200$$

$$20.4X_1 + 17.5X_2 + 24.4X_3 \leq 30$$

$$\mu_1 \leq 1$$

$$\mu_2 \leq 1$$

The solution of the above problem can be obtained using LINGO. The optimal allocation is $X_1 = 0$, $X_2 = 0$, and $X_3 = 0.28$ X and the optimal maximized profit is 2(lakhs).

Conclusion

This study demonstrated the use of multiobjective linear programming problem for solving a production planning problem. It concludes that the formulated fuzzy fractional programming shows how the farmer obtained optimal cropping pattern which maximized total profit with the use of limited resources.

References

- Chadha, S.S. 1993. Dual of the sum of linear and linear fractional program. *European Journal of Operation Research* **67**:136-139.
- Dangwal. R., Sharma. M. K and Singh. P. (2012). Taylor Series Solution of Multiobjective Linear Fractional Programming Problem by Vague Set, *International journal of Fuzzy Mathematics and Systems*, 2(3):245-253.
- Hirche, J. 1984. A note on programming problems with linear plus linear fractional objective functions. *European Journal of Operation Research* **26**(1-2):49-64.
- Jain, S. and Lachhwani, K. 2008. Sum of linear and fractional multiobjective programming problem under Fuzzy rules constraints. *Australian Journal of Basic and Applied Science* **4**(2): 105-108.
- Kumari, P.L., Reddy, G. Kand and Krishna, T.G. (2014). Optimum Allocation of Agricultural Land to the vegetable Crops under uncertain profits using Fuzzy Multiobjective Linear programming. *IOSR journal of Agricultural and Veterinary Sciences*, 7(12):19-28.

- Lone .M. A., Mir. S. A., Singh K.N. and Khan. I. (2015). Linear / Non Linear Plus Fractional Goal Programming (L/NLPFGP) Approach in stratified sampling design. *Research Journal of Mathematical and Statistical Sciences*,3(12): 16-20.
- Lone .M. A., Pukhta. M. S. and Mir. S. A., (2016). Fuzzy Linear Mathematical Programming in Agriculture. *BIBECHANA*, 13: 72-76.
- Narashimann, R.1980. On fuzzy goal programming-some constraints. *Decision Sciences***11**: 532-538.
- Orlovsky, S. A. 1980.On Formulation Of A General Fuzzy Mathematicakl programming problem. *Fuzzy Sets and Systems***3**: 311-321.
- Pal, B.B.,Monitra, B.N. and Maulik, U.2003. A Goal programming procedure for fuzzy multiobjective linear fractional programming problem. *Fuzzy Sets and Systems***139**: 391-405.
- Parra, M.A., Terol, A.B. and Uria, M.V.R. 2001. A Fuzzy Goal Programming Approach to Portfolio selection. *European Journal of Operational Research***133**: 287-297.
- Romero, C. 1991. Handbook of critical issues in goal programming, *Pergamon Press, Oxford*.
- Romero,C and Rehman.T (1989). Multiple criteria Analysis for Agricultural Decisions. Elsevier, Amsterdam.
- Ross, T.J. 1995. Fuzzy logic with engineering Applications. New York: McGraw-Hill.
- Schaible, S. 1977. A note on the sum of linear and linear fractional functions, *Naval Research Logistic Quarterly***24**:961-963.
- Tamiz, M.1996. Multi-Objective programming and goal programming theories and applications. *Germany : Springer-Verlag*
- Zadeh,L.A.1965. Fuzzy sets. *Information and Control* **8**:338-353.
- Zimmermann, H. J.1983. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems***1**:45-55.
- Zimmermann, H.J. 1991. Fuzzy set theory and its applications (2nd rev.ed). Boston: Kulwer.
- Zimmermann, H.J.1978. Using fuzzy sets in Operation research. *European Journal of Operational Research***13**:201-206.