Modelling and Allocation of Crops: Mathematical Programming Approach

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Abstract

Mathematical programming techniques are commonly used by decision makers for achieving efficiency in agricultural production planning. Due to increasing demands of growing population of world, one needs to utilize the limited available resources in the most efficient and economic way. In this paper, the fractional programming problem is formulated and is used to determine the optimal cropping pattern of vegetable crops in such a way that the total profit is maximized. The solution of the formulated Fuzzy programming problem is obtained using LINGO.

Keywords: optimal solution, optimal land allocation, Fractional goal programming, Multiobjective linear programming Problem.

Introduction:

In agricultural field experiments, crop planning is usually carried out to determine which type of crops should be cultivated and the area required for planting the crop. This planning issue is usually solved by using Mathematical programming techniques. Linear programming is one of the oldest techniques of Mathematical programming used for decision making studies. The most ordinary kind of mathematical programming is Fractional programming with objectives [1]. Due to increasing demands of growing population of world the manufacture may have to invest a little more than the initial proposed budget in the interest of his production process. In this situation fuzzy set theory can be used to formulate the model with the help of membership functions. Most of the applications in agricultural planning correspond to the problem of determining an optimum-cropping pattern with multiple goals. Goal Programming techniques have been successfully used for these purposes [2]. Multi-objective linear plus linear fractional programming problem solutions are found in [3, 4, 5, 6, 7, 8] etc. The first mathematical formulation of fuzziness was pioneered by [9]. [10] made a numerous attempts to explore the ability of fuzzy set theory to become a useful tool for adequate mathematical analysis of real world problems. Fuzzy methods have been developed in virtually all branches of decision making problems can be found in [11, 12, 13, 14, 15, 16]. Goal programming approach in fuzzy environment has been first introduced by [17]. Fuzzy goal programming has been discussed by several authors (see [18, 19, 20] etc,).

In this paper we have demonstrated that how a farmer who has limited resources such as availability of labor work time, water and land on which he/she wanted to grow three vegetable crops, Bringal, Tomato and carrot. The farmer's objective is to determine the optimal cropping pattern so that the total profit will be maximized

Linear Fractional Programming

A problem in which the objective function is the ratio of two linear functions and constraints are linear. Such problems are called linear fractional programming problems and can be stated precisely as fallows:

$$\begin{aligned} &Optimize\ Z = \frac{p'x+\alpha}{q'x+\beta} \\ &subject\ to \\ &\qquad Ax = b \\ &\qquad x \geq 0 \end{aligned}$$

where p and q are n vectors, b is an m vector. A is $m \times n$ matrix. α and β are scalars. If an optimal solution for a linear fractional problem exists, then an extreme point optimum exists.

Mathematical Formulation of General Multi-objective Programming Problem

The general multi-objective programming problem with n decision variables, m constraints and p objective is:

$$\left.\begin{aligned} Optimize\ Z &= Z(X_1, X_2, \ldots, X_n) \\ &= [Z_1(X_1, X_2, \ldots, X_n), \\ &\quad Z_2(X_1, X_2, \ldots, X_n), \\ &\qquad \ldots,\ Z_p(X_1, X_2, \ldots, X_n)] \end{aligned}\right\} \qquad (D)$$

$$\begin{aligned} &subject\ to \\ &\qquad g_i(X_1, X_2, \ldots, X_n) \leq 0 \\ &and \quad X_j \geq 0, \quad (i = 1,2,\ldots,m, j = 1,2,\ldots,n) \end{aligned}$$

where, $Z(X_1, X_2, \ldots, X_n)$ is the multi-objective function with $Z_1(X_1, X_2, \ldots, X_n)$, $Z_2(X_1, X_2, \ldots, X_n), \ldots, Z_p(X_1, X_2, \ldots, X_n)$ as p individual objective functions.

For multi-objective linear programming problem (MOLPP) the proposed approach can be outlined as given below:

Step 1: solve problem with each single objective. Here P=3 and Find the minimum value of $MaxZ1, MaxZ2, and\ MaxZ3$, supposing $MaxZ2$, has minimum optimal value.

Step 2: Divide each objective individually say by $MaxZ2$, .

Step 3: we get fractional programming $\xi_1(z1(x)), and\ \xi_2(z2(x))$

Step 4: Define the membership function for P[th] objective.

If $Z_p(x) \leq g_p$ then

$$\mu_p(x)=\begin{cases}1 & if \quad Z_p(x)\le g_{pt}\\ \dfrac{u_p - Z_p(x)}{u_p - g_p} & if \quad g_p \le Z_p(x)\le u_p\\ 0 & if \quad Z_p(x)\ge u_p\end{cases}$$

If $Z_p(x) \ge g_p$ then

$$\mu_t(x)=\begin{cases}1 & if \quad Z_p(x)\ge g_p\\ \dfrac{Z_p(x)-l_p}{g_p - l_p} & if \quad l_p \le Z_p(x)\le g_p\\ 0 & if \quad Z_p(x)\le l_p\end{cases}$$

where g_p is the aspiration level of the p^{th} objective $Z_p(x)$ and u_p and l_p (p= 1, 2 . . . m) are the upper tolerance limit and lower tolerance limit, respectively, for the p^{th} fuzzy goal. [21] presented a fuzzy approach to multi-objective linear programming problems. Now, we formulate the fuzzy programming model of problem (D) by transforming the objective functions into fuzzy goals by assigning aspiration level to each of them using [22] Max-min approach.

Step 5: Now, transform non linear membership functions $\mu_p(x)$ into an equivalent linear membership functions at individual best solution point by using first order Taylor's series as fallows:

$$\mu_p(x)= \mu_p(x_p^*)+[(x_1 - x_{p1}^*)\frac{d\mu_p(x_p^*)}{dx_1}+(x_2 - x_{p2}^*)\frac{d\mu_p(x_{pt}^*)}{dn_2}+. . . +(x_L - x_{tL}^*)\frac{d\mu_p(x_p^*)}{dx_L}]$$

where x_t^* is the individual best solution.

Step 6: Solve the fuzzy goal problem using LINGO.

Numerical Illustration

Suppose a farmer has 8 acres farm on which he/she grow three vegetable crops, Bringal, Tomato and carrot. As per his/her past his expense, the total availability of labor work time , water, seed cost and fertilization cost are 200 (000hrs hours) , 30(acre-inches), 5(in lakhs) and 2 (in lakhs) respectively. The information related to total profit in lakhs obtained from these three crops for one acre of land is given in the tabular form below. Now, the farmer's objective is to determine the optimal cropping pattern so that the total profit will be maximized. The some parts of the example have been taken from [13].

Vegetable crops	Profit (plot1)	Profit (plot2)	Profit (plot3)	Labor requirement (00hrs)	Water requirement/ acre-inches	Seed cost	Fertilization cost
Bringal	1.12	2.10	0.44	1.40	20.4	0.10	1.15

Tomato	0.30	0.40	1.12	1.20	17.5	0.13	0.14
carrot	1.40	0.26	0.86	1.70	24.5	0.12	0.18

Let X_1 be the area required in acres for Bringal crop.
Let X_2 be the area required in acres for Tomato crop and
Let X_3 be the area required in acres for carrot crop.
Therefore, the multi objective problem can be formulated as

$$MaxK1 = 1.12X_1 + 2.10X_2 + 0.44X_3$$
$$MaxK2 = 0.30X_1 + 0.40X_2 + 1.12X_3$$
$$MaxK3 = 1.40X_1 + 0.26X_2 + 0.86X_3$$
$$Subject\ to$$
$$X_1 + X_2 + X_3 \le 8$$
$$1.40X_1 + 1.20X_2 + 1.70X_3 \le 200$$
$$20.4X_1 + 17.5X_2 + 24.5X_3 \le 30$$
$$1.15X_1 + 0.14X_2 + 0.18X_3 \le 2$$
$$0.10X_1 + 0.13X_2 + 0.12X_3 \le 5$$

Using step 1, we get

$$MaxK1 = 3.60, (0,\ 1.71,\ 0)$$
$$MaxK2 = 1.37, (0,\ 0,\ 1.20)$$
$$MaxK3 = 2.05,\ (1.47,\ 0,\ 0)$$

Using step (2 and 3), we have

$$Max\zeta_1$$
$$Max\zeta_2$$
$$Subject\ to$$
$$X_1 + X_2 + X_3 \le 8$$
$$1.40X_1 + 1.20X_2 + 1.70X_3 \le 200$$
$$20.4X_1 + 17.5X_2 + 24.5X_3 \le 30$$
$$1.15X_1 + 0.14X_2 + 0.18X_3 \le 2$$
$$0.10X_1 + 0.13X_2 + 0.12X_3 \le 5$$

After solving this we get

$$Max\zeta_1 = 5.25,\ (0,\ 0.4,\ 0)$$
$$Max\zeta_2 = 4.47, (0.32,\ 0,\ 0)$$

Using step 4 and 5, we have

$$\zeta_1(x) = -0.29X_1 - 34X_3 + 5.25$$
$$\zeta_2(x) = 6.51X_2 - 39.78X_3 + 4.47$$

Thus the fractional programming problem is now transferred in linear programming. The fuzzy goal programming is as follows with their possible aspiration levels as given below:

$$\zeta_1(x) = -0.29X_1 - 34X_3 + 5.25 \leq 5.25$$
$$\zeta_2(x) = 6.51X_2 - 39.78X_3 + 4.47 \leq 4.47$$
Subject to
$$X_1 + X_2 + X_3 \leq 8$$
$$1.40X_1 + 1.20X_2 + 1.70X_3 \leq 200$$
$$20.4X_1 + 17.5X_2 + 24.5X_3 \leq 30$$
$$1.15X_1 + 0.14X_2 + 0.18X_3 \leq 2$$
$$0.10X_1 + 0.13X_2 + 0.12X_3 \leq 5$$

Let (6, 5) be the tolerance limits for two goals respectively. The membership function can be defined for both of the two goals as

$$\mu_1(x) = \begin{cases} 1 & if \quad \zeta_1(x) \leq 5.25 \\ \dfrac{6-\zeta_1(x)}{0.75} & if \quad 5.25 \leq \zeta_1(x) \leq 6 \\ 0 & if \; \zeta_1(x) \geq 6 \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & if \quad \zeta_2(x) \leq 4.47 \\ \dfrac{\zeta_2(x)-5}{0.53} & if \quad 4.47 \leq \zeta_2(x) \leq 5 \\ 0 & if \; \zeta_2(x) \geq 5 \end{cases}$$

Now, using step 6, Fuzzy goal programming can be formulated as

$$Max\, G = \mu_1 + \mu_2$$
Subject to
$$0.75\mu_1 - 0.29X_1 - 34X_3 = 0.75$$
$$-0.53\mu_2 + 6.51X_2 - 39.78X_3 = 0.53$$
$$X_1 + X_2 + X_3 \leq 8$$
$$1.40X_1 + 1.20X_2 + 1.70X_3 \leq 200$$
$$20.4X_1 + 17.5X_2 + 24.5X_3 \leq 30$$
$$1.15X_1 + 0.14X_2 + 0.18X_3 \leq 2$$
$$0.10X_1 + 0.13X_2 + 0.12X_3 \leq 5$$
$$\mu_1 \leq 1$$
$$\mu_2 \leq 1$$

The solution of the above problem can be obtained using LINGO. The optimal allocation is $X_1 = 0, X_2 = 0, and\ X_3 = 0.16$ and the optimal maximized profit is 2 (lakhs).

Conclusion

This study demonstrated the use of multi-objective linear programming problem for solving a production planning problem. It concludes that the formulated fuzzy fractional programming shows how the farmer obtained optimal cropping pattern which maximized total profit with the use of limited resources.

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