Path Connectedness over Soft Rough Topological Space

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***Abstract*—In this paper, we mainly discuss the path connectedness of the soft rough topological space. We study the properties of connected soft rough real space, give the definition of path between soft points, and discuss the property of path connectedness in the soft rough topological space, and the relation between connected soft rough topological space and path local connected soft rough topological space.**

Keywords: soft rough formal context, connectedness, connected soft rough topological space, soft point, path connected space.

1. Introduction

Formal concept context[1,2] , rough set[2,4], soft set[5,6] are extensively applied in the uncertainty reasoning, and many authors have discussed the related issues in these fields, and produced many meaningful research results mostly listed the reference ([7]-[16]). Topology[17] is also applied to these fields as one of the greatest unifying ideas of mathematics listed in the reference [18]-[23].

In [23], the authors defined isolated subset and demonstrated the properties of isolated subset. At the same time, in this paper, the soft rough connected topological space is defined, and connectedness of soft rough topological space is discussed. In [24], the author defined the connected branch and further discussed the local connectedness over the connected soft rough topological space. However, the above researches were carried out without referring path connectedness.

Therefore, in this paper, based on the connected soft rough topological space which author proposed in [23], we further discuss path connectedness of the soft topological space over the soft rough formal context . The rest of this paper is organized as following: in section 2, we review some basic concepts and properties of rough concept formal and soft sets. In section 3, we give the properties of the continuous mapping between the soft rough topological spaces and discuss the connected soft rough real topological space under the continuous mapping. In section 4, we define path between soft points and path connectedness of given soft rough topological space, and discuss the properties of path connectedness. In short, conclusions are given in section 5.

2. **Basic knowledge**

Definition 2.1. [5] Let U be an initial universe set and E be a set of parameters. Let $\mathcal{P}(U)$ denotes the power set of U and $A \subset E$. Then a pair (F, A) is called a **soft set** over U, where $F : A \to \mathcal{P}(U)$ is a mapping.

That is, the soft set is a parameterized family of subsets of the set U. Every set $F(e), \forall e \in E$, from this family may be considered as the set of e-elements of the soft set (F, E), or considered as the set of e-approximate elements of the soft set.

According to this manner, we can view a soft set (F, E) as consisting of collection of approximations: $(F, E) = \{F(e) \mid e \in E\} = \{(F(e), e) \mid e \in M\}$.

Definition 2.2. [16] Let (G, M, R) is a rough formal context, G is objects set, is also called the universe, M is attributes set. A pair (F, B) is a soft set over G, where $B \subseteq M$, and $F : B \to \mathcal{P}(G)$ is a set-value mapping over G, furthermore, the lower and upper rough approximations of pair (F, B) are denoted by $\underline{R}(F, B) = (\underline{F}, B), \overline{R}(F, B) = (\overline{F}, B)$, which are soft sets over G with the set-valued mappings given by $\underline{F}(x) = \underline{B}(F(x))$ and $\overline{F}(x) = \overline{B}(F(x))$, where $x \in B$. The operators $\underline{R}, \overline{R}$ are called **the lower and upper rough approximation operators** on soft set (F, B).

If $\overline{R} = \underline{R}$, we say that the soft set (F, B) is **definable**, otherwise, (F, B) is **rough**.

we call such quadruple tuple (G, M, R, F) as **rough soft formal context**, and, such soft set (F, B) on the rough soft formal context (G, M, R, F) which is called **rough soft set**.

Obviously, $\forall x \in B \subseteq M, F(x) \subseteq G$ is a parameterized family of subsets of G, and $F(x)$ is the set of x- approximate elements in (G, M, R, F).

Remark 2.1. *In this definition, we first give the soft set, then discuss their rough properties, so we call it as rough soft formal context. However, if we first discuss the rough properties, then find the soft set, and we call it as soft rough formal context. In this paper, our study is based on soft rough formal context.*

Definition 2.3. [16] Let (G, M, R, F) be a soft rough formal context over the objects set G, and attributes set M. $B_1, B_2 \subseteq M$, (F_1, B_1) and (F_2, B_2) are two soft sets $F_i : B \to \mathcal{P}(G), i = 1, 2$ is a set-value mapping over G on (G, M, R, F).
(i) If $B_1 \subseteq B_2$, and $F_1(x) \subseteq F_2(x), \forall x \in B_1 \subseteq B_2$, then (F_1, B_1) is a **soft subset** of (F_2, B_2), denoted as $(F_1, B_1)\widetilde{\subset}(F_2, B_2)$.
(ii) (F_1, B_1) and (F_2, B_2) are said **soft equal** , if $(F_1, B_1)\widetilde{\subset}(F_2, B_2)$, and $(F_2, B_2)\widetilde{\subset}(F_1, B_1)$. We simply denote by $(F_1, B_1) = (F_2, B_2)$.

Definition 2.4. [16] Let $F : B \to \mathcal{P}(G)$, and (F, B) be a soft rough set over the soft rough formal context(G, M, R, F), we define the soft complement as:

(1) The **relative complement** of (F, B) is denoted by $(F, B)^c$ and is defined by $(F, B)^c = (F^c, B)$, where $F^c : B \to \mathcal{P}(G)$ and $F^c(x) = G - F(x), \forall x \in B$.

Clearly, $((F, B)^c)^c = (F, B)$.

(2) (F, B) is said to be a **relative null soft rough set** denoted by $\mathcal{N}$, if $\forall x \in B, F(x) = \emptyset$, if $B = M$, then is called **absolute null soft rough set**, denoted as $\widetilde{\emptyset}$.

(3) (F, B) is said to be a **relative whole soft rough set** denoted by $\widetilde{G}$, if $\forall x \in B, F(x) = G$.

Definition 2.5. [16] Let (G, M, R, F) be the soft rough formal context, $F_1, F_2 : B \to \mathcal{P}(G)$ are two set-value mapping over G on (G, M, R, F). (F_1, B_1) and (F_2, B_2) are two soft rough sets over (G, M, R, F).

(i) The **union** of (F_1, B_1) and (F_2, B_2) is the soft rough set (H, C), where $C = B_1 \cup B_2$, and $\forall e \in C$, denoted as $(F_1, B_1)\widetilde{\cup}(F_2, B_2) = (H, C) = (H, B_1 \cup B_2)$, where

$$H(e) = \begin{cases} F_1(e), & if \quad e \in B_1 - B_2 \\ F_2(e), & if \quad e \in B_2 - B_1 \\ F_1(e) \cup F_2(e), & if \quad e \in B_1 \cap B_2 \end{cases}$$

(ii) The **intersection** of (F_1, B_1) and (F_2, B_2) is the soft rough set (H, C) is denoted as $(F_1, B_1) \cap (F_2, B_2)$ and is defined as $(F_1, B_1) \sqcap (F_2, B_2) = (H, C)$, where $C = B_1 \cap B_2$, and $\forall e \in C, H(e) = F_1(e) \cap F_2(e)$.

Definition 2.6. [21] Let $\mathcal{T} = (G, M, R, F)$ be a soft rough formal context over the object set G and attributes set M, $B_i \subseteq M$, $\tau = \{(F_i, B_i) \mid (F_i, B_i)$ is a soft set over $G\}$ which is the collection of soft sets on the soft rough formal context (G, M, R, F), if

(1) $\widetilde{\emptyset}, \widetilde{G}$ belong to τ.

(2) The union of any number of soft sets in τ belongs to τ, that is, τ is closed for the any union of soft sets over $\mathcal{T}$.

(3) The intersection of any two soft sets in τ belongs to τ, that is, τ is closed for the finite intersection of soft sets over $\mathcal{T}$.

Then the collection τ is calleda **soft topology** over $\mathcal{T}$(simply called **soft rough topology**). The triplet (G, τ, M) is called **a soft topological space** over $\mathcal{T}$ (simply called **soft rough topological space**), and the members of τ are **soft open sets** in $\mathcal{T}$, the relative complement $(F, B)^c = (F^c, B)$ is said to be a **soft closed set** in $\mathcal{T}$ if $(F, B)^c \in \tau$.

If (F, B) is both soft open and soft closed, then (F, B) is a **soft clopen set**.

Definition 2.7. [23] Let (G, τ, M) be a soft topological space over $\mathcal{T}$, and (F, B) be a soft set in τ, and $x \in G$. we say $x \in (F, B)$(read as x **belongs to** the soft set (F, B)) if $x \in F(e)$ for all $e \in B$, and if there is some $e \in B$, $x \notin F(e), \forall x \in G$, then $x \notin (F, B)$. The points belong to some soft set, we call these points as **soft points**.

Definition 2.8. [23] Let the triplet (G, τ, M) be a soft rough topological space over $\mathcal{T}$, $Y \subseteq G, (F_i, M), i = 1, 2$ are soft sets, $\widetilde{Y}$ be a soft open (closed) subsets of $\widetilde{G}$.

(i) if $(F_1, M), (F_2, M)\widetilde{\subset}\widetilde{G}$ are soft sets in τ, where $((F_1, M) \sqcap \overline{(F_2, M)})\widetilde{\cup}((F_2, M) \sqcap \overline{(F_1, M)}) = \widetilde{\emptyset}$, then soft sets (F_1, M) and (F_2, M) are **isolated soft subsets** of $\widetilde{G}$, simply, (F_1, M) and (F_2, M) are **isolated**.

(ii) if $\widetilde{G} = (F_1, M)\widetilde{\cup}(F_2, M)$, then (G, τ, M) **is not connected**, otherwise, (G, τ, M) is **connected**.

(iii) If $(Y, \tau \mid_Y, M)$ is a soft rough subspace of (G, τ, M) which is a connected , then $\widetilde{Y}$ is **a connected soft subset** of $\widetilde{G}$, otherwise, $\widetilde{Y}$ **is not a connected** soft subset of $\widetilde{G}$.

(iv) If $\exists(F, B) \in \tau, \forall e \in B, F(e) \subseteq G$ such that $x \in (F, B)$ and $y \in (F, B)$, then x and y are connected.

Proposition 2.1. *[23] Let $(\mathbb{R}, \tau, M)$ be a soft rough real topological space, in which $\tau = \{(F, B) \mid \forall e \in B \subseteq M, F(e) \subseteq \mathbb{R}\}$, then $(\mathbb{R}, \tau, M)$ is a connected.*

Corollary 2.1. *Any interval E of $\mathbb{R}$, if $(E, \tau|_E, M)$ is a soft rough topological subspace of $(\mathbb{R}, \tau, M)$, then $(E, \tau|_E, M)$ is soft connected.*

Conversely , we have

Proposition 2.2. *Let (G, τ, M) be a soft topological space, $Y_\gamma \subseteq G, \gamma \in \Gamma$, Γ is an index set, $\{\widetilde{Y_\gamma}\}_{\gamma\in\Gamma}$ is a class of connected soft subset of $\widetilde{G}$, if $\sqcap_{\gamma\in\Gamma}\widetilde{Y_\gamma} \neq \emptyset$, then $\widetilde{\cup}_{\gamma\in\Gamma}\widetilde{Y_\gamma}$ is a connected soft subset of $\widetilde{G}$.*

Proof: Suppose that $(F_1, M), (F_2, M)$ are isolated soft subsets of $\widetilde{G}$, and $\widetilde{\cup}_{\gamma\in\Gamma}\widetilde{Y_\gamma} = (F_1, M)\widetilde{\cup}(F_2, M)$. For any $x \in \sqcap_{\gamma\in\Gamma}\widetilde{Y_\gamma}$, without loss of the generality, assume that $x \in (F_1, M)$, by $\widetilde{Y_\gamma}$ being connected, having $\widetilde{Y_\gamma}\widetilde{\subset}(F_1, M)$ or $\widetilde{Y_\gamma}\widetilde{\subset}(F_2, M)$, so $\sqcap_{\gamma\in\Gamma}\widetilde{Y_\gamma}\widetilde{\subset}(F_1, M)$ and $(F_2, M) = \widetilde{\emptyset}$ which infers $\widetilde{\cup}_{\gamma\in\Gamma}\widetilde{Y_\gamma}$ is connected. ■

Proposition 2.3. *Let (G, τ, M) be a soft topological space, $Y \subseteq G, \forall x, y \in Y$, there exists a connected soft subset $\widetilde{Y_{xy}}$ of $\widetilde{G}$, such that $x, y \in \widetilde{Y_{xy}}\widetilde{\subset}\widetilde{Y}$, then $\widetilde{Y}$ is a connected soft subset of $\widetilde{G}$.*

Proof:

Case 1. *If $Y = \emptyset$, then $\widetilde{Y}$ is clearly a connected soft subset of $\widetilde{G}$.*

Case 2. *If $Y \neq \emptyset$, then $\forall a \in Y$, $\widetilde{Y} = \widetilde{\cup}_{y\in Y}\widetilde{Y_{ay}}$, and $a \in \sqcap_{y\in Y}\widetilde{Y_{ay}}$, by the above proposition, we can know that $\widetilde{Y}$ is a connected soft subset of $\widetilde{G}$.*

■

3. THE PROPERTIES OF SOFT REAL REAL TOPOLOGICAL SPACE

In the following, (G, τ, M) is a soft rough topological space and $B \subseteq M$, (F, B) is a soft set in τ.

To avoid repetition, we do not write it again. As the description in [23], we have known the soft rough real topological space is connected, next , we study some properties of this connected soft rough real topology.

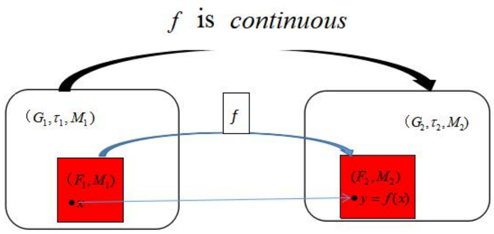

Fig. 1.

Assuming that the reader is familiar with soft set, topology, soft rough formal context, and soft connectedness, we review the basic knowledge used in this paper and more information can see [19-23].

Definition 3.1. [23] Let (G_1, τ_1, M_1) and (G_2, τ_2, M_2) be soft rough topological spaces, $f : (G_1, \tau_1, M_1) \longrightarrow (G_2, \tau_2, M_2)$ is a mapping. if $\forall (F_2, B_2) \in \tau_2, \exists (F_1, B_1) \in \tau_1$, such that $f(F_1, B_1) = (F_2, B_2)(\forall e_2 \in B_2, \exists e_1 \in B_1$, such that $f(F_1(e_1)) = F_2(e_2),$), that is: the pre-image of soft open set is soft open. Then f is a **continuous mapping**.

Remark 3.1. *If (G_1, τ_1, M_1) and (G_2, τ_2, M_2) are connected soft rough topological spaces, $f : (G_1, \tau_1, M_1) \longrightarrow (G_2, \tau_2, M_2)$ is a continuous mapping, then $\forall y \in G_2, y$ must belongs to a soft set $(F_2, M_2) \in \tau_2$, similarly, $x \in G_1, x$ must belongs to a soft set $(F_1, M_1) \in \tau_1$, so , we can describe this continuous mapping f using different methods:*

(1) For universe, $\forall x \in G_1$, $\exists y = f(x) \in G_2$ is the image of x, and denote the image of the universe G_1 as $f(G_1)$ which is the subset of G_2. Furthermore, $\forall x \in G_1$, because (G_1, τ_1, M_1) is a soft rough topological space, x must be a soft point in (G_1, τ_1, M_1)(that is , there is at least one soft set $(F_1, B_1) \in \tau_1$, such that $x \in (F_1, B_1) \in \tau_1$) , $\exists y \in (F_2, B_2) \in \tau_2$, having $f(x) = y$ under f (see Fig. 1).

Hence, in the following, $x \in G$ means x is a soft point in $(G.\tau, M)$.

(2) For soft set, $\forall (F_1, B_1) \in \tau_1, \exists (F_2, B_2) \in \tau_2$, such that $f(F_1, B_1) = (F_2, B_2)$.

So, for convenience, we can simply denote the image of (G_1, τ_1, M_1) as $f(G_1)$, and

$$\begin{aligned} f(G_1) &= \{f(F_1(e_1)|e_1 \in M_1, x \in F_1(e_1) \subseteq G_1\} \\ &= \{F_2(e_2)|e_2 \in M_2, y \in F_2(e_2) \subseteq G_2\} \\ &= \{y \in G_2 | y \in F_2(e_2) \subseteq G_2, f(x) = y,\ x \in G_1\} \end{aligned}$$

For example, if $f : (G, \tau_1, M_1) \to (\mathbb{R}, \tau_2, M_2)$ is a continuous mapping, then for all $x \in G$, the the image of x is a real number, and and $\exists (F_2, B_2) \in \tau_2 = \{(F_2, B_2)| F_2 : B_2 \to \mathbb{R}, \forall e_2 \in B_2 \subseteq M_2, F(e_2) \subseteq \mathbb{R}\}$, such that $f(x) \in (F_2, B_2)$.

Theorem 3.1. *Let (G, τ, M) and (G', τ', M') be soft rough topological spaces, $\exists (F_1, B_1), (F_2, B_2) \in \tau$, s.t. $(F_1, B_1)\widetilde{\cup}(F_2, B_2) = \widetilde{G}$. denote: $A = \bigcup_{e \in B_1} F_1(e), B = \bigcup_{e \in B_2} F_2(e)$. Let $f_1 : (A, \tau|_A, M) \longrightarrow (G', \tau', M')$ and $f_2 : (B, \tau|_B, M) \longrightarrow (G', \tau', M')$ be continuous mappings, and $\forall x \in G$, if $x \in A \cap B$, then $f_1(x) = f_2(x)$. Define $f : (G, \tau, M) \longrightarrow (G', \tau', M')$ such that $\forall x \in G$,*

$$f(x) = \begin{cases} f_1(x), & if \quad x \in A \\ f_2(x), & if \quad x \in B \end{cases}$$

then f is continuous.

Proof: $\forall Y \subseteq G'$, then $\widetilde{Y}$ is soft subset of $\widetilde{G'}$, and $f_1^{-1}(\widetilde{Y}) = f^{-1}(\widetilde{Y}) \sqcap \widetilde{A}$, $f_2^{-1}(\widetilde{Y}) = f^{-1}(\widetilde{Y}) \sqcap \widetilde{A}$, so $f^{-1}(\widetilde{Y}) = f_1^{-1}(\widetilde{Y})\widetilde{\cup}f_2^{-1}(\widetilde{Y})$.

Suppose that (F', B') is any soft open set of τ', then $f_1^{-1}(F', B') \in \tau|_A \subseteq \tau$, $f_2^{-1}(F', B') \in \tau|_B \subseteq \tau$, so, $f^{-1}(F', B') = f_1^{-1}(F', B')\widetilde{\cup}f_2^{-1}(F', B')$, that is, the pre-image of soft open set is also open. And

$$\left.\begin{array}{l} f_1^{-1}(\widetilde{Y}) = f^{-1}(\widetilde{Y}) \sqcap \widetilde{A} \\ f_2^{-1}(\widetilde{Y}) = f^{-1}(\widetilde{Y}) \sqcap \widetilde{A} \end{array}\right\} \implies f^{-1}(\widetilde{Y}) = f_1^{-1}(\widetilde{Y})\widetilde{\cup}f_2^{-1}(\widetilde{Y})$$

Hence, f is continuous.

■

Remark 3.2. *(1) This theorem is* **Pasting Lemma(Glueing Lemma)** *in topology.*

(2) This result is also true, if (F_1, B_1) and (F_2, B_2) are soft closed sets .

Theorem 3.2. [23] *Let $f : (G_1, \tau_1, M_1) \longrightarrow (G_2, \tau_2, M_2)$ be a continuous mapping, if (G_1, τ_1, M_1) is soft connected, then $f(\widetilde{G_1})$ is a soft connected subset of $\widetilde{G_2}$.*

This theorem tells us that the connectedness is invariant under the continuous mapping.

Proposition 3.1. *Let $(\mathbb{R}, \tau, M)$ be a soft rough real topological space, $\widetilde{E}$ is a connected subset of $\widetilde{\mathbb{R}}$, that is, (E, τ_E, M) is a connected soft rough topological subspace of $(\mathbb{R}, \tau, M)$, then E is an interval.*

Proof: Suppose that E is not an interval, then $\exists a, b \in E$, without loss the generality, assume that $a < b$ such that the interval $[a, b] \nsubseteq E$ which means $\exists c \in [a, b], a < c < b$, and $c \notin E$. Denote $A = (-\infty, c) \cap E, B = (c, +\infty) \cap E$, obviously, A and B are non-empty open subset of E, and $\widetilde{A} \cup \widetilde{B} = \widetilde{E}, \widetilde{A} \cap \widetilde{B} = \emptyset$, then $\widetilde{E}$ is not connected, which is contradict with $\widetilde{E}$ being a connected soft subset of $\widetilde{\mathbb{R}}$.

Hence, E is an interval. ■

Corollary 3.1. *Let (G, τ_1, M_1) be a connected soft rough topological space, $f : (G, \tau_1, M_1) \longrightarrow (\mathbb{R}, \tau_2, M_2)$ be a continuous mapping, then*

(1) $f(G)$ is an interval.

(2) For any $x, y \in G$, $f(x)$ and $f(y)$ which are real numbers are their images under the continuous mapping f, t is any real number between $f(x)$ and $f(y)$, then $\exists z \in G$ such that $f(z) = t$.

Remark 3.3. *Because (G, τ_1, M_1) is connected, $\forall x, y \in G, \exists (F_1, B_1)\widetilde{\subset}\widetilde{G}, (F_2, B_2)\widetilde{\subset}\widetilde{G}$ such that $x \in (F_1, B_1)$, and*

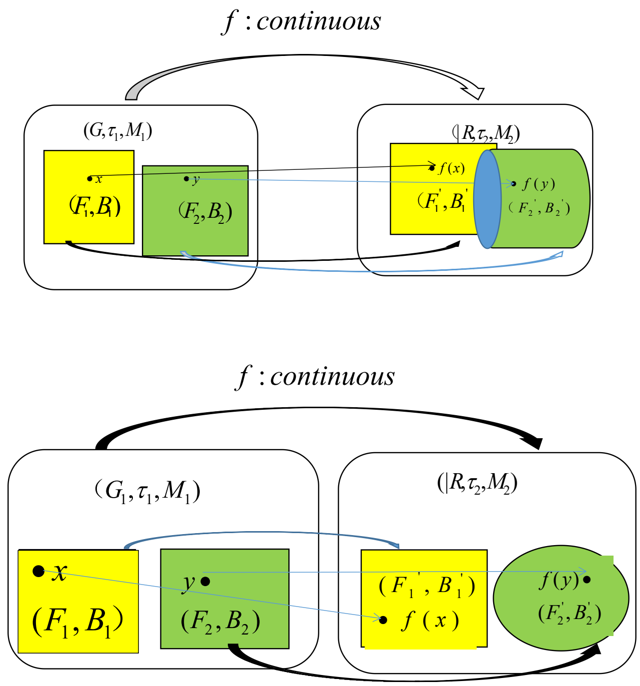

Fig. 2.

$y \in (F_2, B_2)$, (if x and y are soft connected, then$(F_1, B_1) = (F_2, B_2)$). Denote the images of $(F_1, B_1), (F_2, B_2)$ respectively as $(F_1', B_1'), (F_2', B_2')$, that is, $f(F_1, B_1) = (F_1', B_1')$ $f(F_2, B_2) = (F_2', B_2')$. $f(x)$ and $f(y)$ are the images of x and y respectively.

Because $f(x), f(y)$ are real numbers, $f(x) > f(y), f(x) = f(y), f(x) < f(y)$ must have one and only one held by the law of the excluded middle of real numbers.

However, (F_1', B_1') and (F_2', B_2') need not have any inclusion relations, that is, it is not necessary for $(F_1', B_1')\widetilde{\subset}(F_2', B_2')$ or $(F_2', B_2')\widetilde{\subset}(F_1', B_1')$, in other words , they can be $(F_1', B_1')\sqcap(F_2', B_2') = \widetilde{\emptyset}$ or $(F_1', B_1')\sqcap(F_2', B_2') \neq \widetilde{\emptyset}$ (see Fig. 2).

Proof: (1) It is obvious from the above proposition.

(2) The conclusion hods if $f(x) = f(y)$. Next, suppose that $f(x) \neq f(y)$, without loss of the generality, assume that $f(x) < f(y)$, then $[f(x), f(y)] \subseteq F(G) \subseteq \mathbb{R}$, and $f(G)$ is an interval which infers $\forall t \in \mathbb{R}, f(x) \leq t \leq f(y)$ having $t \in f(G)$.

Hence, $\exists z \in G$ such that $f(z) = t$. ■

Theorem 3.3. (Intermediate value theorem)

Let $f : ([a, b], \tau_1, M_1) \longrightarrow (\mathbb{R}, \tau_2, M_2)$ be a continuous mapping, $f(x)$ and $f(y) \in \mathbb{R}$ be the images points of x, y under f($\forall x, y \in [a, b]$), then for any real numbert between $f(x)$ and $f(y)$, there is a real number z in $[a, b](i.e.z \in [a, b])$ such that $f(z) = t$.

Proof: The proof is easy using the above proposition. ■

Remark 3.4. *Because $([a, b], \tau_1, M_1)$ and $(\mathbb{R}, \tau_2, M_2)$ are connected, $\tau_1 = \{(F_1, B_1)|\forall e_1 \in B_1 \subseteq M_1, F_1(e_1) \subseteq B_1 \subseteq M_1\}$ and $\widetilde{\cup}_{(F_1,B_1)\in\tau_1}(F_1, B_1) = \widetilde{[a, b]}$, for any $\forall x \in [a, b]$, there is at least one soft set $(F_1, B_1) \in \tau_1$, such that $x \in (F_1, B_1)$ which means any point x in $[a, b]$ is a soft point, similarly, any point is also a soft point in $\mathbb{R}$.*

4. The Path Connected of the Soft Rough Topological Space

Definition 4.1. Let (G, τ, M) be a soft rough topological space, x, y be soft points in (G, τ, M).

(1) If there is a continuous mapping f from $([a, b], \tau|_{[a,b]}, M)([a, b] \subseteq \mathbb{R})$ to (G, τ, M), such that $f(a) = x, f(b) = y$, then f is a **path from the soft point** x **to the soft point** y, simply , f is a **path from** x **to** y.

(2) If there is path form x to y in (G, τ, M) , then we say x and y is path connected.

(3) If for any pair (x, y) of soft points in (G, τ, M), there is a path from x to y, then we say (G, τ, M) is a **path connected soft rough topological space**, to be short, (G, τ, M) is **path connected**.

Remark 4.1.

(1) For simplicity, we take interval $[a, b]$ as unit closed interval $[0, 1]$.

(2) The relation of path connectedness is an equivalent relation.

Proof: For any soft points x, y, z in $G, \tau, M)$, we have:

(i) x and x is path connected.

In fact, let $f : ([0, 1], \tau|_{[0,1]}, M) \to (G, \tau, M)$, such that $f(t) = x, \forall t \in [0, 1]$, then f is a path from x to x.

(ii) If x and y is path connected, then y and x is also path connected.

In fact, suppose $f : ([0, 1], \tau|_{[0,1]}, M) \to (G, \tau, M)$ is a path from x to y, define $f'(t) = f(1 - t), \forall t \in [0, 1]$, then f' is a path from y to x.

(iii) If x and y is path connected, y and z is path connected, then x and z is also path connected.

In fact, suppose that $f_1, f_2 : ([0, 1], \tau|_{[0,1]}, M) \to (G, \tau, M)$ are paths from x to y and from y to z, respectively, define mapping $f : ([0, 1], \tau|_{[0,1]}, M) \to (G, \tau, M)$, such that ,

$$f(t) = \begin{cases} f_1(2t), & if \quad t \in [0, \frac{1}{2}] \\ f_2(2t - 1), & if \quad x \in [\frac{1}{2}, 1] \end{cases} \qquad \forall t \in [0, 1]$$

then f is continuous by Pasting Lemma, and $f(0) = f_1(0) = x, f(1) = f_2(1) = z$.

Hence, f is a path from x to z. ■

Example 4.1. $(\mathbb{R}, \tau, M)$ is path connected.

In fact, we know $(\mathbb{R}, \tau, M)$ is connected, so any point x in $(\mathbb{R}, \tau, M)$ is a soft point, that is , $\exists(F, B) \in \tau$, such that $x \in (F, B)$. Define continuous mapping f as $f : ([0, 1], \tau|_{[0,1]}, M) \longrightarrow (\mathbb{R}, \tau, M)$, such that, $\forall(F', B') \in \tau|_{[0,1]}, \exists(F, B) \in \tau$, having $f(F', B') = (F, B) \in \tau$, and $\forall t \in [0, 1], \exists x, y \in \mathbb{R}$, $f(t) = x + t(y - x) \in \mathbb{R}$(**Note**: x, y and t are soft points), clearly, $f(t)$ is a soft point in $\mathbb{R}$, that is , there is at least one soft set $(F, B) \in \tau$, such that $f(t) \in (F, B)$. Then f is a path from x to y.

Furthermore, we know $[0,1]$ and $\mathbb{R}$ have the same cardinal number, so for any pair of soft point (x, y), this continuous mapping will be a path from x to y.

Hence, $(\mathbb{R}, \tau, M)$ is path connected.

For example, let $M = \{e_1, e_2, e_3\}, \tau = \{(F, B)|F : B \rightarrow I \subseteq \mathbb{R}, I$ is any interval $\}$. Take $B_1 = \{e_1, e_3\}, B_2 = \{e_2\}$, and $x = \frac{1}{2}(F_1, B_1) = \{(e_1, [0, 1]), (e_3, (-1, 3]\}$, $y = 2(F_2, B_2) = \{(e_2, (\frac{2}{3}, 3)\}, \forall t \in [0, 1], \exists x, y \in \mathbb{R}$, such that $f(t) = \frac{1}{2} + \frac{2}{3}t \in \mathbb{R}$, there is at least one interval $I \subseteq \mathbb{R}$, such that $f(t) \in I$, without loss of the generality, suppose that $I = [a, b] \subseteq [0, 1]$ and let $f(a) = x = \frac{1}{2},\ f(b) = 2$, so $f(t)$ is a path from $\frac{1}{2}$ to 2 in $(\mathbb{R}, \tau, M)$.

Theorem 4.1. *Let (G, τ, M) be a path connected soft rough topological spaces, then (G, τ, M) is connected.*

Proof: Because (G, τ, M) is path connected, for any $x, y \in G$, there is a path $f : ([0, 1], \tau|_{[0,1]}, M) \longrightarrow (G, \tau, M)$ from(soft point) x to (soft point) y which is continuous mapping, and we have known that $([0, 1], \tau|_{[0,1]}, M)$ is connect, so its image $f([0, 1], \tau|_{[0,1]}, M)$ is also connected (see [23]), that is $f(\widetilde{[0, 1]})$ is a connected soft subset of $\widetilde{G}$.

Hence, (G, τ, M) is connected. ■

Remark 4.2. *The converse of this theorem is not true, in other words, if (G, τ, M) is connected , then (G, τ, M) can not be path connected.*

However, we can prove:

Theorem 4.2. *Let $A \subseteq \mathbb{R}$, $\widetilde{A}$ is a connected soft (open) subset of $\widetilde{\mathbb{R}}$, then $(A, \tau|_A, M)$ is connected if and only if $(A, \tau|_A, M)$ is path connected.*

Proof: Suppose that $A \subseteq \mathbb{R}$ and $(A, \tau|_A, M)$ is path connected, then $(A, \tau|_A, M)$ is connected by above theorem.

Conversely, assume that $(A, \tau|_A, M)$ is connected, if $|A| = 1$, *i.e.* A is singleton, then A is clearly path connected. If $|A| > 1$, then A is an interval of $\mathbb{R}$, and $(A, \tau|_A, M)$ is path connected by Proposition 3.1. ■

Theorem 4.3. *Let (G, τ, M) and (G', τ', M') be soft rough topological spaces, $f : (G, \tau, M) \longrightarrow (G', \tau', M')$ be a continuous mapping from (G, τ, M) to $G', \tau', M')$. If (G, τ, M) is path connected, then its image $(f(G), \tau'|_{f(G)}, M')$ is also path connected.*

Proof: $\forall y_1, y_2, \exists (F_1', B_1'), (F_2', B_2') \in \tau'$, such that $y_1 \in (F_1', B_1')$, $y_2 \in (F_2', B_2')$, take $x_1 \in (F_1, B_1)$, $x_2 \in (F_2, B_2) \in \tau$, such that $f(x_1) = y_1, f(x_2) = y_2$. Because (G, τ, M) is path connected, there is a path $g : ([0, 1], \tau|_{[0,1]}, M) \longrightarrow (G, \tau, M)$ define a mapping $h : ([0, 1], \tau|_{[0,1]}, M) \longrightarrow (f(G), \tau'|_{f(G)}, M')$, such that $\forall t \in [0, 1], h)t) = f \circ g(t)$, so, h is a path from y_1 to y_2. Hence, $(f(G), \tau'|_{f(G)}, M')$ is path connected. ■

5. Conclusion

In this paper, the issue discussed is the path connectedness of the soft rough topological space based on the soft connected rough topological space. Studying the properties of connected soft rough real space, we give the definition of path between soft points, and at the same time, we discuss the property of path connectedness in the soft rough topological space, and the relation between connected soft rough topological space and path local connected soft rough topological space, which will offer a brand-new idea in data analysis.

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