Original Research Article

Inventory Model for Three–Parameter Weibull Deterioration and Partial Backlogging

ABSTRACT

This paper develops an economic order quantity inventory model for time dependent three parameters Weibull deterioration. Partially backlogged shortages are considered. The demand rate is deterministic and time dependent. The rate of deterioration is time dependent. We have derived the most favorable order quantity model by minimizing the entire inventory cost. A numerical illustration has been carried out to evaluate the result of parameters on decision variables and the total average cost of the model. The research focus of this paper is to derive the optimum order quantity by minimizing the total inventory cost.

Keywords: Weibull Deterioration, Constant Demand, Inventory, Partial Backlogging

1. INTRODUCTION

Inventory model is much affected by the deterioration. This is defined as change, spoilage, and decay in the number of items during storage period. The cause of deterioration is extremely significant for most of the goods which cannot be ignored in inventory system. The fundamental economic order quantity model considers a stable demand rate, infinite scheduling horizon, deterioration of inventory and insignificant lead time. These presumptions delimitate the utility of the traditional economic order quantity inventory model. Some examples of items like fashion items, foods, electronics items like air conditioner, washing machine, mobile, press and heater, drugs in which deterioration take place during storage period and this must be account when analyzing the inventory model.

Definition: A random variable t is said to have a Weibull distribution with parameters $\alpha > 0, \beta > 0, \gamma > 0$, if the probability density function of t is given by $f(t) = \alpha\beta(t - \gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}}, t > 0$. The parameter α is the scale parameter, β is the shape parameter and γ is the location parameter. If $\gamma < 0$, then it indicates that deterioration has occurs prior to the starting of the manufacture.

Ghare and Schrader [7] developed the inventory model considering the effect of deterioration. In this paper they considered constant rate of deterioration with no shortage. Shah and Shah [14], Goyal and Giri [8] presented review on deterioration inventory model. Covert and Philip [4] presented an inventory model where the time to deterioration is described with two parameter Weibull deterioration. Ghosh and Chaudhuri [9] developed an inventory model for two parameter Weibull deteriorating items, with shortages and quadratic demand rate. Sanni [16] developed an inventory model for three parameter Weibull deteriorating items, with shortages and quadratic demand rate. Wu and Ouyang [17] developed an inventory model by considering two types of shortages in the model.

Model that starts with stock and starts with shortages is obtained optimal replenishment policy for the different cases. Samanta and Bhowmick [15] considered two parameter Weibull distribution to represent the time to deterioration and allowed shortages in the inventory. They studied two cases; where the inventory starts with shortages and the case where the inventory starts without shortages and derived the economic order quantity for the respective system.

This paper develops an inventory model for time dependent three parameters Weibull deterioration. Partially backlogged shortages are considered. The demand rate is deterministic and time dependent. The research focus of this paper is to present a mathematical model for the system in which we present the most favorable order strategy for the model and set up the necessary and sufficient conditions for the optimal policy.

The Weibull distribution deterioration is appropriate for goods whose rate of deterioration is raise with time and the location parameter γ in the three parameters Weibull distribution. We have derived the most favorable order quantity model by minimizing the entire inventory cost. A numerical illustration has been carried out to evaluate the result of parameters on decision variables and the total inventory cost of this model.

2. ASSUMPTIONS AND NOTATIONS

These parameters are considered to develop the mathematical model:

2.1 Assumptions:

1. The inventory model is developed for single item.
2. The demand rate $P(t)$ is known and constant.
3. Replenishment rate is infinite.
4. Insignificant lead time.
5. Partially backlogged shortages are considered.
6. The deterioration rate, $\theta(t) = \alpha\beta(t-\gamma)^{\beta-1}$, follows a three parameter Weibull distribution; where α is the scale parameter, $(0 < \alpha \leq 1)$; β is the shape parameter, $\beta > 0$ and γ is the location parameter, $\gamma > 0$.
7. The deterioration increases with time $t > 0$.
8. The backlogging rate during the shortage period is variable and depends on the lead time till the next replenishment. The partial backlogging rate will be $B(t) = \frac{1}{1+\Phi(T-t)}; where \Phi > 0$ is the backlogging parameter and $t_1 \leq t \leq T$.

2.2 Notations:

O_c: Per order ordering cost.
C_p: Per unit purchasing cost.
h_c: Per unit time holding cost.
h_b: Per unit per time unit backorder cost.
h_1: Per unit cost of lost sales.
$P(t)$: Demand rate at any time $t \geq 0$.

T: Cycle length i.e. $T = t_1 + t_2$.

t_1: The inventory level reaches to zero at that time $t_1 \geq 0$.

t_2: Shortages are allowed in this period $t_2 \geq 0$.

q_m: Size of maximum inventory during $(0, T)$.

q_b: During stock out period maximum backordered units occur

R: During a cycle length T, total order quantity occurs i.e. $R = q_m + q_b$.

$q_1(t)$: Positive inventory at time t, $0 \leq t \leq t_1$.

$q_2(t)$: Negative inventory at time t, $t_1 \leq t \leq T$.

TAC: Per time unit total average cost.

3. MATHEMATICAL MODEL

At the initial stage of the cycle, the inventory level reaches its maximum q_m units of item at time $t = 0$. The inventory depletes due to demand and partially to deterioration during $[0, t_1]$. The inventory level reaches to zero at time $t = t_1$. The rate of deterioration is defined by an increasing function of time $\theta(t) = \alpha\beta(t-\gamma)^{\beta-1}$.

The inventory system $q_1(t)$ is defined by the following differential equation

$$\frac{dq_1(t)}{dt} + \theta q_1(t) = -P, 0 \leq t \leq t_1$$

Putting the value of θ, we get

$$\frac{dq_1(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1} q_1(t) = -P, 0 \leq t \leq t_1 \qquad (3.1),$$

with boundary conditions, $q_1(t_1) = 0, q_1(0) = q_m, 0 \leq t \leq t_1$

The solution of equation (3.1) is

$$q_1(t) = P \begin{bmatrix} t_1 - t + \frac{\alpha}{\beta+1}\left((t_1-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}\right) \\ -\alpha t_1 (t-\gamma)^{\beta} + \alpha t (t-\gamma)^{\beta} \end{bmatrix}, 0 \leq t \leq t_1 \qquad (3.2)$$

The maximum inventory becomes

$$q_m(0) = Pt_1 + \frac{\alpha}{\beta+1}\left((t_1-\gamma)^{\beta+1} — \gamma\right)\beta + 1 - \alpha t1 - \gamma\beta \qquad (3.3)$$

Inventory level depletes and reaches to zero at time t_1, after that shortages are occurred. Inventory level depends on demand and a part of the demand is partially backlogged during the interval $[t_1, T]$.

The position of inventory can be defined by the following differential equation:

$$\frac{dq_2(t)}{dt} = \frac{-P}{1+\Phi(T-t)}, t_1 \leq t \leq T \qquad (3.4)$$

with boundary conditions, $q_2(t_1) = 0 t_1 \leq t \leq T$

The solution of equation (3.4) is

$$q_2(t) = \frac{P}{\Phi}\left[ln\left(1 + \Phi(T-t)\right) - ln(1 + \Phi t_2)\right], t_1 \leq t \leq T$$

(3.5)

The maximum backordered units become

$$q_b(t) = -q_2(t_1 + t_2) = \frac{P}{\Phi}[ln(1 + \Phi t_2)], t_1 \leq t \leq T$$

(3.6)

The order size R during $[0, T]$ becomes $R = q_m + q_b$

$$R = P\left[t_1 + \frac{\alpha}{\beta+1}\left((t_1-\gamma)^{\beta+1} - (-\gamma)^{\beta+1}\right) - \alpha t_1 (-\gamma)^{\beta} + \frac{1}{\Phi} ln(1 + \Phi t_2)\right]$$

(3.7)

To compute the total average cost per unit time, we need the following components:

$$TAC = \frac{1}{t_1 + t_2}[OC + HC + BC + LS + PC]$$

Per cycle ordering cost (OC); $OC = O_c$

Per cycle holding cost during $[0, t_1]$is

$$HC = h_c \int_0^{t_1} q_1(t)dt$$

$$= h_c P\left[\frac{{t_1}^2}{2} + \frac{\alpha}{\beta+1}\left\{\left(\frac{2(-\gamma)^{\beta+2}}{\beta+2} - \frac{2(t_1-\gamma)^{\beta+2}}{\beta+2}\right) + t_1\left((-\gamma)^{\beta+1} + (t_1-\gamma)^{\beta+1}\right)\right\}\right]$$

Per cycle backorder cost is

$$BC = h_b \int_{t_1}^{t_1+t_2} -q_2(t)dt = h_b \frac{P}{\Phi^2}[\Phi t_2 - ln(1+\Phi t_2)]$$

Per cycle cost of lost sales is

$$LS = h_1 P \int_{t_1}^{t_1+t_2}\left(1 - \frac{1}{1+\Phi(t_1+t_2-t)}\right)dt = h_1\frac{P}{\Phi}[\Phi t_2 - ln(1+\Phi t_2)]$$

Per cycle purchasing cost is

$$PC = C_p Q = C_p P\left[t_1 + \frac{\alpha}{\beta+1}\left((t_1-\gamma)^{\beta+1} - (-\gamma)^{\beta+1}\right) - \alpha t_1(-\gamma)^{\beta} + \frac{1}{\Phi}ln(1+\Phi t_2)\right]$$

Per unit time total average cost is

$TAC = \frac{1}{t_1+t_2}[OC + HC + BC + LS + PC]$

We have, $TAC = \frac{1}{t_1+t_2}\left[\begin{array}{l} O_c - \frac{\alpha C_p P(-\gamma)^{\beta+1}}{\beta+1} + \frac{2\alpha h_c P(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + P\left(C_p - \alpha C_p(-\gamma)^{\beta} + \frac{\alpha h_c(-\gamma)^{\beta+1}}{\beta+1}\right)t_1 \\ + \frac{h_c P}{2}{t_1}^2 - \frac{2\alpha h_c P}{(\beta+1)(\beta+2)}(t_1-\gamma)^{\beta+2} + \frac{\alpha C_p P}{\beta+1}(t_1-\gamma)^{\beta+1} + \frac{\alpha h_c P}{\beta+1}(t_1-\gamma)^{\beta+1}t_1 \\ + P\left(h_1 + \frac{h_b}{\Phi}\right)t_2 + P\left(\frac{C_p}{\Phi} - \frac{h_1}{\Phi} - \frac{h_b}{\Phi^2}\right)ln(1+\Phi t_2) \end{array}\right]$

$$TAC = \frac{1}{t_1+t_2}\left(\begin{array}{l} A_1 + B_1 t_1 + C_1{t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 + G_1 t_2 \\ + H_1\, ln(1+\Phi t_2) \end{array}\right)$$

(3.8)

where

$A_1 = O_c - \frac{\alpha C_p P(-\gamma)^{\beta+1}}{\beta+1} + \frac{2\alpha h_c P(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}$, $B_1 = P\left(C_p - \alpha C_p(-\gamma)^{\beta} + \frac{\alpha h_c(-\gamma)^{\beta+1}}{\beta+1}\right)$,

$C_1 = \frac{h_c P}{2}$, $D_1 = \frac{2\alpha h_c P}{(\beta+1)(\beta+2)}$, $E_1 = \frac{\alpha C_p P}{\beta+1}$, $F_1 = \frac{\alpha h_c P}{\beta+1}$, $G_1 = P\left(h_1 + \frac{h_b}{\Phi}\right)$, $H_1 = P\left(\frac{C_p}{\Phi} - \frac{h_1}{\Phi} - \frac{h_b}{\Phi^2}\right)$

From equation (3.8), we get

$$\frac{\partial TAC}{\partial t_1} = \frac{1}{t_1+t_2}\left(\begin{array}{l} B_1 + 2C_1 t_1 - D_1(\beta+2)(t_1-\gamma)^{\beta+1} + E_1(\beta+1)(t_1-\gamma)^{\beta} + F_1(t_1-\gamma)^{\beta+1} \\ + F_1(\beta+1)(t_1-\gamma)^{\beta}t_1 \end{array}\right) - \frac{1}{(t_1+t_2)^2}\left(\begin{array}{l} A_1 + B_1 t_1 + C_1{t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 + G_1 t_2 \\ + H_1\, ln(1+\Phi t_2) \end{array}\right)$$

(3.9)

$$\frac{\partial TAC}{\partial t_2} = \frac{1}{t_1+t_2}\left(G_1 + \frac{\delta H_1}{1+\Phi t_2}\right) - \frac{1}{(t_1+t_2)^2}\Big(A_1 + B_1 t_1 + C_1 {t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 + G_1 t_2 + H_1\, ln(1+\Phi t_2)\Big)$$

(3.10)

$$\frac{\partial^2 TAC}{\partial {t_1}^2} = \frac{1}{t_1+t_2}\begin{pmatrix} 2C_1 - D_1(\beta+1)(\beta+2)(t_1-\gamma)^{\beta} + E_1\beta(\beta+1)(t_1-\gamma)^{\beta-1} \\ +F_1(\beta+1)(t_1-\gamma)^{\beta} + F_1(\beta+1)(t_1-\gamma)^{\beta} + F_1\beta(\beta+1)(t_1-\gamma)^{\beta-1}t_1 \end{pmatrix} - \frac{2}{(t_1+t_2)^2}\begin{pmatrix} B_1 + 2C_1 t_1 - D_1(\beta+2)(t_1-\gamma)^{\beta+1} + E_1(\beta+1)(t_1-\gamma)^{\beta} + F_1(t_1-\gamma)^{\beta+1} \\ +F_1(\beta+1)(t_1-\gamma)^{\beta}t_1 \end{pmatrix} + \frac{2}{(t_1+t_2)^3}\begin{pmatrix} A_1 + B_1 t_1 + C_1 {t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 \\ +G_1 t_2 + H_1\, ln(1+\Phi t_2) \end{pmatrix}$$

$$\frac{\partial^2 TAC}{\partial t_2 \partial t_1} = -\frac{1}{(t_1+t_2)^2}\begin{pmatrix} B_1 + 2C_1 t_1 - D_1(\beta+2)(t_1-\gamma)^{\beta+1} + E_1(\beta+1)(t_1-\gamma)^{\beta} + F_1(t_1-\gamma)^{\beta+1} \\ +F_1(\beta+1)(t_1-\gamma)^{\beta}t_1 + G_1 + \frac{\delta H_1}{1+\Phi t_2} \end{pmatrix} + \frac{2}{(t_1+t_2)^3}\begin{pmatrix} A_1 + B_1 t_1 + C_1 {t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 + G_1 t_2 \\ +H_1\, ln(1+\Phi t_2) \end{pmatrix}$$

(3.11)

$$\frac{\partial^2 TAC}{\partial {t_2}^2} = -\frac{1}{t_1+t_2}\left(\frac{\Phi^2 H_1}{(1+\Phi t_2)^2}\right) - \frac{2}{(t_1+t_2)^2}\left(G_1 + \frac{\Phi H_1}{1+\Phi t_2}\right) + \frac{2}{(t_1+t_2)^3}\Big(A_1 + B_1 t_1 + C_1 {t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 + G_1 t_2 + H_1\, ln(1+\Phi t_2)\Big)$$

(3.12)

$$\frac{\partial^2 TAC}{\partial t_1 \partial t_2} = -\frac{1}{(t_1+t_2)^2}\begin{pmatrix} G_1 + \frac{\Phi H_1}{1+\Phi t_2} + B_1 + 2C_1 t_1 - D_1(\beta+2)(t_1-\gamma)^{\beta+1} + E_1(\beta+1)(t_1-\gamma)^{\beta} \\ +F_1(t_1-\gamma)^{\beta+1} + F_1(\beta+1)(t_1-\gamma)^{\beta}t_1 \end{pmatrix} + \frac{2}{(t_1+t_2)^3}\Big(A_1 + B_1 t_1 + C_1 {t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 + G_1 t_2 + H_1\, ln(1+\Phi t_2)\Big)$$

(3.13)

To minimize the total average cost, it is proved that

$\frac{\partial^2 TAC}{\partial {t_1}^2} \times \frac{\partial^2 TAC}{\partial {t_2}^2} - \frac{\partial^2 TAC}{\partial t_1 \partial t_2} > 0$

The necessary condition $\frac{\partial^2 TAC}{\partial {t_1}^2} \times \frac{\partial^2 TAC}{\partial {t_2}^2} - \frac{\partial^2 TAC}{\partial t_1 \partial t_2} > 0$is satisfied.

To find the values of (t_1, t_2), it is necessary that $\frac{\partial TAC}{\partial t_1} = 0$ and $\frac{\partial TAC}{\partial t_2} = 0$.

Now, we have

$$(t_1+t_2)\begin{pmatrix} B_1 + 2C_1 t_1 - D_1(\beta+2)(t_1-\gamma)^{\beta+1} + E_1(\beta+1)(t_1-\gamma)^{\beta} + F_1(t_1-\gamma)^{\beta+1} \\ +F_1(\beta+1)(t_1-\gamma)^{\beta}t_1 \end{pmatrix} - \Big(A_1 + B_1 t_1 + C_1 {t_1}^2 - D_1(t_1-\gamma)^{\beta+2} + E_1(t_1-\gamma)^{\beta+1} + F_1(t_1-\gamma)^{\beta+1}t_1 + G_1 t_2 + H_1\, ln(1+\Phi t_2)\Big) = 0$$

(3.14)

$$(t_1 + t_2)\left(G_1 + \frac{\Phi H_1}{1 + \Phi t_2}\right)$$
$$-\left(A_1 + B_1 t_1 + C_1 {t_1}^2 - D_1 (t_1 - \gamma)^{\beta+2} + E_1 (t_1 - \gamma)^{\beta+1} + F_1 (t_1 - \gamma)^{\beta+1} t_1 + G_1 t_2 + H_1\, ln(1 + \Phi t_2)\right) = 0$$

(3.15)

Solving these equations, we get the values of t_1 and t_2 which minimize the total average cost $TAC(t_1, t_2)$ provided that they satisfied the following sufficient conditions:
The sufficient condition is:
$k_1 > 0$, $k_4 > 0$, $k_1.k_4 - k_2.k_3 > 0$.
If $k_2 = k_3$, the condition reduces to: $k_1 > 0$, $k_4 > 0$, $k_1.k_4 - (k_2)^2 > 0$,
where $k_1 = \frac{\partial^2 TAC}{\partial {t_1}^2}$, $k_2 = \frac{\partial^2 TAC}{\partial t_1 \partial t_2}$, $k_3 = \frac{\partial^2 TAC}{\partial t_2 \partial t_1}$and $k_4 = \frac{\partial^2 TAC}{\partial {t_2}^2}$respectively.

4. NUMERICAL EXAMPLE

Example 4.1:
The example presents an inventory system with the following data:

$$O_c = 300, C_p = 10, h_c = 0.7, h_b = 32, h_1 = 20, P = 2000, \Phi = 2, \alpha = 0.3, \beta = 2, \gamma = 0.6$$

Putting these values in the mathematical model and we get the optimum values of $t_1 \to 1.5$ and $t_2 \to 1.23$. Putting the values of t_1 and t_2 in the equation (3.7), and we get the optimum value of order size $R = 4106$. Now, putting the values of t_1 and $t_2$in the equation (3.8), and we get minimum value of total average cost $TAC = 237055$.
Example 4.2:
The example presents an inventory system with the following data:

$$O_c = 200, C_p = 12, h_c = 0.9, h_b = 36, h_1 = 22, P = 1000, \Phi = 2, \alpha = 0.1, \beta = 1, \gamma = 0.4$$

Putting these values in the mathematical model and we get the optimum values of $t_1 = 1.18$ and $t_2 = 0.27$. Putting the values of t_1 and $t_2$in the equation (3.7), and we get the optimum value of order size $R = 1465$. Now, putting the values of t_1 and $t_2$in the equation (3.8), and we get minimum value of total average cost $TAC = 13821.67$.

5. CONCLUSION

In this paper, deterioration is considered in inventory decision making. A deterministic inventory model is derived for time dependent three parameter Weibull deterioration. Partially backlogged shortages are considered. The research focus of this paper is to derive the optimum order quantity by minimizing the total inventory cost. The unsatisfied demand is time dependent and backlogged. It is important to set up the necessary and sufficient conditions for the optimal solution. A numerical example is proposed to illustrate the mathematical model. It is observed that increase in scale or shape or location parameter and demand parameter result is increase in order quantity and total inventory cost. Further research carried out such as finite replenishment, exponential demand, for multi items and no quantity discount.

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