# Modelling the effect of Hartmann Number on Transient period, Viscous dissipation and Joule heating in a Transient MHD flow over a flat plate moving at a constant velocity

B. M. Nyamai

*Department of Mathematics and Physics, Moi University, P.O Box 3900, Eldoret, Kenya*
*Corresponding Author Email: nyamaib@ymail.com*

**ABSTRACT**

This study is designed to investigate the effect of Hartmann number on transient period, Joule heating and viscous dissipation in an incompressible MHD (Magneto-Hydrodynamics) flow over a flat plate moving at a constant velocity. The governing momentum equation is non-dimensionalized and solved by the Laplace transform technique. The solution is decomposed into transient part and steady state part and then the effect of Hartmann number on transient period concerning velocity and its two related quantities (Joule heating and viscous dissipation) is analyzed. It was found out that when Hartmann number is increased the transient period is shortened and it was the same for the three quantities. In addition, the steady state solutions for both Joule heating and viscous heating were found to be equal. Even though velocity decreases when the Hartmann number is increased, the opposite was discovered for both Joule heating and viscous heating. Graphical analysis indicated that transient period changes considerably if Hartmann number is between 0 and 2. This study will find use in those industrial areas where magnetic fields are used to control liquid / molten metals in open channels.

**Keywords:** MHD, Hartmann number, transient period, Joule and Viscous dissipation.

**NOMENCLATURE**

| | |
|---|---|
| $B$ | Magnetic field $(Wb/m^2)$ |
| $D$ | viscous dissipation variable |
| $E_c$ | Eckert number |
| erfc | complementary error function |
| erf | error function |
| H | Hartmann number |
| $J$ | Joule heating variable |
| MHD | Magneto-Hydrodynamics |
| U | plate velocity $(m/s)$ |
| t | time parameter $(s)$ |
| u | x component of velocity $(m/s)$ |
| v | y component of velocity $(m/s)$ |
| $\nu$ | kinematic viscosity $(m^2/s)$ |
| $\rho$ | density $(kg/m^3)$ |
| $\sigma$ | electrical conductivity $(W/(m.K))$ |

**superscripts**

| | |
|---|---|
| $*$ | dimensionless variable |

**subscripts**

| | |
|---|---|
| $p$ | permanent |
| $t$ | transient |

# 1. INTRODUCTION

MHD flow concerns with the flow of electrically conducting fluids in the presence of magnetic fields. These type of flows where introduced by Hannes Alfven. This made him to win a Nobel prize in 1970.

Research in MHD flow has attracted the attention of many researchers due to its numerous industrial and engineering applications. For instance, they find application in metal processing, metallurgical work, welding, MHD ship propulsion, EM pumps, flow meters and MHD flow conducting pumps among others. The description of these applications is found in Tillack and Morley (1998). Further, they find applications in geophysics, earthquake studies, astrophysics and cosmology and nuclear cooling. Magnetic drug targeting is another potential area of application. A discussion of these applications has been done in http:en.wikipedia.org/wiki/magnetohy-drodynamics.

The literature pertaining unsteady and steady MHD flows is huge. The time after which an unsteady flow becomes steady is called transient period or time. This period is important especially in engineering application since it is used to determine or predict the time which must elapse in order for steady state conditions to be assumed. The current study therefore aims at investigating the effect of the Hartmann number on transient period of velocity and its related quantities; viscous dissipation and Joule heating.

The effect of Hartmann number on MHD flow on various quantities of interest has been investigated by many researchers. Mamalonkas (2001) investigated, using the finite difference method MHD flow over an oscillating plane. The results indicated that Hartmann number reduces the fluid velocity but increases the fluid temperature. Makinde and Mhone (2005), analytically, studied the combined effect of transient magnetic field and radiation in a channel field with porous medium. It was discovered that magnetic strength intensity reduces wall shear stress. Chaudhary and Jain (2007) applied the Laplace transform technique to study the combined heat and mass transfer effects on MHD free convection flow past a plate. This study showed that skin friction increases with an increase in Hartmann number which is contrary to Makinde and Mhone (2005). Further it was demonstrated graphically that an increase in the Hartmann number increases the fluid temperature and decreases velocity. The effect of thermal conductivity on unsteady MHD flow over a semi-infinitive vertical plate research was conducted by Loganathan et al (2010). In this study, the method of Crank – Nicolson type was applied. It was observed that temperature increases with an increase in Hartmann number but skin friction decreases with an increase with Hartmann number. Unsteady MHD free convection flow past a vertical flat plate is done by Hamad and Rop (2011). The perturbation technique was used in this study. It was found that an increase in Hartmann number leads to an increase in skin friction as well as the temperature of the fluid. However, the velocity decreases due to the increased Lorentz force. The effects of thermal radiation and viscous dissipation on MHD heat were investigated by Kishore et al. (2012) using the explicitly finite difference method of Dufort – Frankel's type. It was shown that an increase in Hartmann number increases the skin friction which is due to the enhanced Lorentz force.

Many applications are given in this paper and a rich literature of transient flows is found therein. Ahmed and Kalita (2013) analytically and numerically using the Laplace transform technique and finite differences of the Crank-Nicolson method respectively analyzed MHD flow of an incompressible flow over a vertical oscillating plate in a porous medium in the presence of homogeneous chemical reaction. Through graphical presentation as well as tabular form, it was shown that velocity increases when magnetic parameter was increased. Recently, Abdulhameed et al. (2016) researched on transient MHD flow generated by periodic wall using the He's homotopy perturbation technique method. Just like the previous authors, it was found that an increase in Hartmann number caused a decrease in velocity and an increase in temperature. Thankal (2017) studied unsteady MHD flow over porous stretching plate. Using similarity transformation, it was observed that an increase in Hartmann number increases the skin friction as well as heat transfer rate.

The above literature review reveals the effect of Hartmann number on velocity, temperature, local skin friction as well as local rate of heat transfer. However, its effect on transient period of velocity, viscous and Joule dissipation is lacking. Further its effect on viscous dissipation and Joule heating has not been investigated. This study therefore intends to investigate its effects on transient period, viscous dissipation and Joule heating. We shall attempt to answer the questions:

1) Does an increase in Hartmann number shorten or lengthen transient period of velocity, viscous dissipation and Joule heating?
2) Is the time taken to reach steady state the same for velocity, viscous dissipation and Joule heating?
3) Does an increase in Hartmann number lead to heating up of the fluid through viscous dissipation and Joule heating?

## 2 MATHEMATICAL FORMULATION

Consider an electrically conducting incompressible flow caused by a flat plate moving at a constant speed U. Further, let a magnetic field of strength B be applied perpendicularly to the plate immediately after the plate starts moving. Assume that y is the perpendicular distance away from the plate surface (see fig.1). Let the leading edge of the plate be at x = 0. Assuming a no-slip condition, the governing equations are:

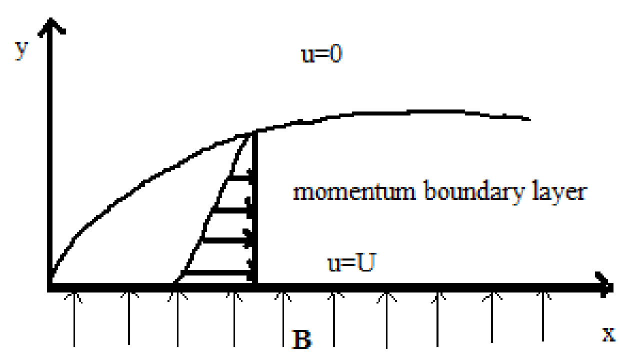


**Fig.1 Physical configuration**

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B^2 u}{\rho}$$

(1)

Subject to

$$u = 0 \text{ at } t \geq 0$$

$$u(0,t) = U \text{ for } t \geq 0 \quad (2)$$

$$u(\infty,t) = 0 \text{ for } t \geq 0$$

In order to non-dimensionalize Eq. (1) and Eq. (2) we shall introduce the dimensionless variables:

$$t* = \frac{tU^2}{\nu}, \quad y* = \frac{yU}{\nu}, \quad u* = \frac{u}{U}, \quad H^2 = \frac{\sigma B^2 \nu}{\rho U^2} \quad (3)$$

In view of this, Eq. (1) and Eq. (2) reduces to Eq. (4) and Eq.(5) respectively.

$$\frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2 \nu u^*}{\rho U^2} \quad (4)$$

Subject to

$$u^*(y^*,t^*) = 0 \quad at \quad t^* = 0$$

$$u^*(0,t^*) = 1 \text{ at } y^* = 0 \quad for \quad t^* \geq 0 \quad (5)$$

$$u^*(\infty,t^*) = 0 \quad for \quad t^* \geq 0$$

For the purpose of clarity, the asterisk ($*$) is dropped in the next sections.

## 3. METHOD OF SOLUTION

Taking the Laplace transform of Eq. (4) and Eq. (5) yields

$$\frac{dU(y,s)}{dy} = -(s + H^2)U(y,s) \quad (6)$$

Which should be solved subject to

$$U(0,s) = \frac{1}{s} \qquad y = 0 \qquad \text{for all} \quad t \geq 0$$

$$U(\infty,s) = 0 \qquad \text{for all} \qquad t \geq 0 \quad (7)$$

The general solution of Eq. (6) is

$$U(y,s) = A\exp(y\sqrt{s + H^2}) + B\exp(-y\sqrt{s + H^2}) \quad (8)$$

Applying the boundary conditions as in Eq. (7) we find

$A = 0$ and $B = \frac{1}{s}$.

Hence Eq. (8) reduces to

$$U(y,s) = \frac{1}{s}\exp(-y\sqrt{s + H^2}) \quad (9)$$

Inverting Eq. (9) yields

$$u(y,t) = \frac{1}{2}\left[\exp(yH)erfc\left(\eta + \sqrt{H^2 t}\right) + \exp(-yH)erfc\left(\eta - \sqrt{H^2 t}\right)\right] \quad (10)$$

This can be expressed in terms of the error function as

$$u(y,t) = \frac{1}{2}\exp(yH)\left(1 - erf\left(\eta + \sqrt{H^2 t}\right)\right) + \frac{1}{2}\exp(-yH)\left(1 - erf\left(\eta - \sqrt{H^2 t}\right)\right) \quad (11)$$

The solution can be written as

$$u(y,t) = u_t(y,t) + u_p(y,t)$$

whereby

$$u_t(y,t) = \frac{1}{2}\exp(yH)\left(1 - erf\left(\eta + \sqrt{H^2 t}\right)\right) \quad (12)$$

$$u_p(y,t) = \frac{1}{2}\exp(-yH)\left(1 - erf\left(\eta - \sqrt{H^2 t}\right)\right) \quad (13)$$

Now as $t \to \infty$, $u_t \to 0$ and $u_p \to \exp(-yH)$.

$u_t$ is called the transient part of the solution and $u_p$ the steady state (permanent) part. It is worth noting that $\exp(-yH)$ is the solution of the steady state momentum equation. That is Eq. (4) when $\frac{\partial u}{\partial t} = 0$.

## 4. VISCOUS DISSIPATION

The effect of viscous shear forces generate heat. This process is called viscous dissipation. The mathematical formula is

$$D(y,t) = E_c\left(\frac{\partial u}{\partial y}\right)^2 \quad (14)$$

In the light of this study we have

$$D(y,t)=\frac{E_c}{4}\Big[H\Big(\exp(yH)\big(1-erf\big(\eta+\sqrt{H^2t}\big)\big)- \exp(-yH)\big(1-erf\big(\eta-\sqrt{H^2t}\big)\big)\Big)-$$

$$\frac{2}{\sqrt{2\pi}}\left(\left(\exp\left(-\left(\eta+\sqrt{H^2t}\right)^2+yH\right)\right)+\left(\exp\left(-\left(\eta-\sqrt{H^2t}\right)^2-yH\right)\right)\right)\Big]^2 \qquad (15)$$

$D(y,t)$ can be decomposed into a transient part $D_t(y,t)$ and permanent part $D_P(y,t)$ as follows

$$D_t(y,t)=\frac{E_c}{4}\Big[H\exp(yH)\big(1-erf\big(\eta+\sqrt{H^2t}\big)\big)\text{-}\ \frac{2}{\sqrt{2t}}\left(\left(\exp\left(-\left(\eta+\sqrt{H^2t}\right)^2+yH\right)\right)+\right.$$

$$\left.\left(\exp\left(-\left(\eta-\sqrt{H^2t}\right)^2-yH\right)\right)\right)\Big]^2 \qquad (16)$$

$$D_p(y,t)=\frac{E_c}{4}\Big[-H\exp(-yH)\big(1-erf\big(\eta-\sqrt{H^2t}\big)\big)\Big]^2 \qquad (17)$$

## 5. JOULE HEATING

Joule heating is the process by which the flow of electric current through the MHD fluid produces heat and is expressed mathematically as

$$J(y,t)=E_cH^2u^2$$

$$J(y,t)=\frac{E_cH^2}{4}\Big[\exp(yH)\big(1-erf\big(\eta+\sqrt{H^2t}\big)\big)+\exp(-yH)\big(1-erf\big(\eta-\sqrt{H^2t}\big)\big)\Big]^2 \qquad (18)$$

Here the transient part of the solution is

$$J_t(y,t)=\frac{E_cH^2}{4}\Big[\exp(yH)\big(1-erf\big(\eta+\sqrt{H^2t}\big)\big)\Big]^2 \qquad (19)$$

and the permanent part is

$$J_p(y,t)=\frac{E_cH^2}{4}\Big[\exp(-yH)\big(1-erf\big(\eta-\sqrt{H^2t}\big)\big)\Big]^2 \qquad (20)$$

## 6. RESULTS AND DISCUSSION

In order to get a true picture of the effect of the Hartmann number (H) on transient period of various quantities we need to analyze the above solutions. The permanent solutions of both viscous heating and joule heating are obtained by letting t tend to infinity. Those effects of H on various quantities which could not be established by analysis of solutions were presented graphically.

### 6.1 Analysis of solutions

It can be seen from equation Eq. (12) that the time taken for velocity to transit from transient state to steady state decreases when the Hartmann number ($H$) is increased. From Eq. (16) we find that $D_t(y,t) \to 0$ as $H \to \infty$. This indicates that it takes shorter time to reach steady state viscous dissipation. In other words, the bigger the Hartmann number the shorter the transient period. Equation (19) reveals that $J_t(y,t) \to 0$ as $H \to \infty$. This means that the bigger the Hartmann number the shorter the transient period.

From Eq. (17) and Eq. (20) we find that as

$t \to 0$, $D_p(y,t) \to H^2 \exp(-2yH)$

and

$J_p(y,t) \to H^2 \exp(-2yH)$ respectively.

It therefore means that the steady state Joule heating and viscous heating are equal which is an interesting discovery.

### 6.2 Graphical analysis

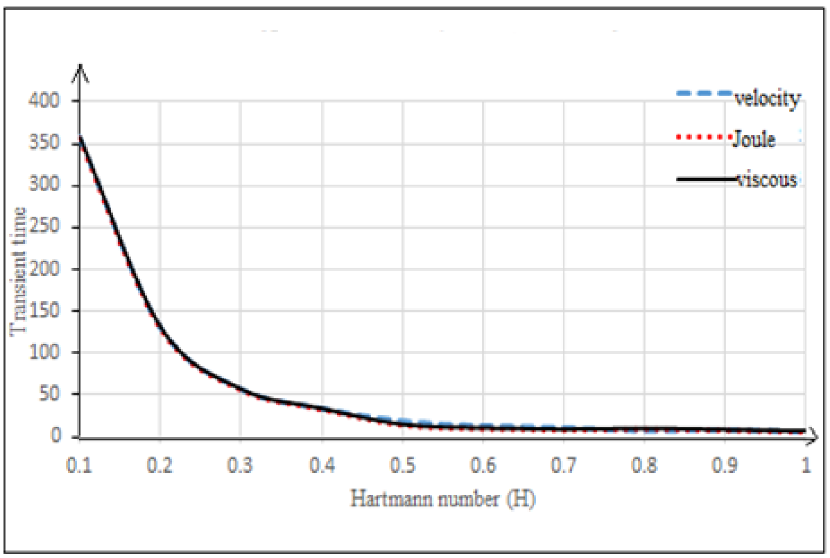


**Fig.2 (a) The effect of Hartmann number on Transient period**

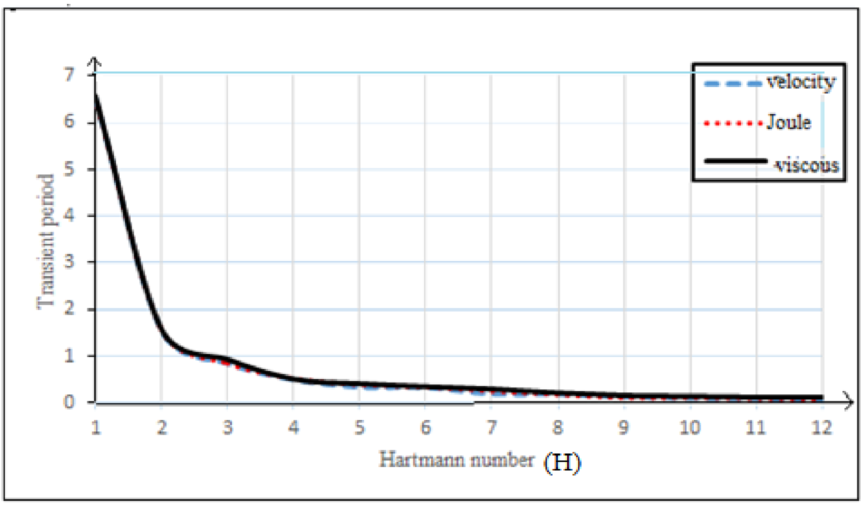


**Fig.2 (b) The effect of Hartmann number on Transient period**

Figure 2 (a) and Fig. 2(b) are graphs of transient period against Hartmann number H. Figure 2 (a) considers values of H between 0.1 and 1 and Fig. 2(b) those between 1 and 12. We can therefore take the second graph as a continuation of the first one. The graph drops sharply between H=0.1 and H=2. After which it drops gradually to almost zero at about H=10. This shows that a small change in H between 0.1 and 2 shortens the transient period significantly and a huge change in H above 2 causes an insignificant change of the same. Generally, the transient period decreases when H is increased. Further, the three curves coincide meaning that the steady state velocity, Joule heating and viscous dissipation are reached at the same time.

Fig. 3 exhibits the effects of the Hartmann number on viscous dissipation. Obviously, as the Hartmann number is increased viscous dissipation increases. When the Hartmann number increases, the velocity decreases. This makes the relative velocity between the fluid and the moving plate huge. In turn the velocity gradient increases hence the increase in viscous heating.

Fig. 4 displays the effect of the Hartmann number on Joule heating. It is observed that as the Hartmann number increases Joule heating increases. The big emf produced when Hartmann number is increased leads to a large current flow.  In turn more heat is dissipated.

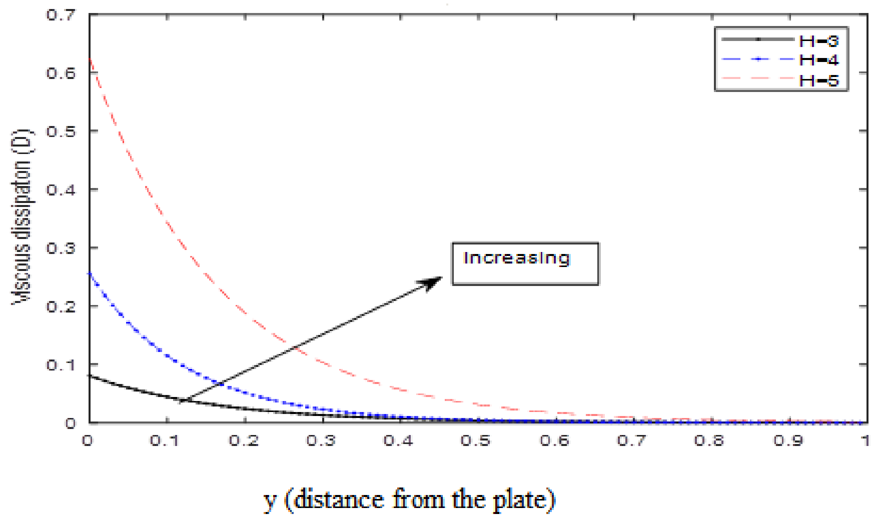


**Fig. 3 the effect of Hartmann Number on viscous dissipation**

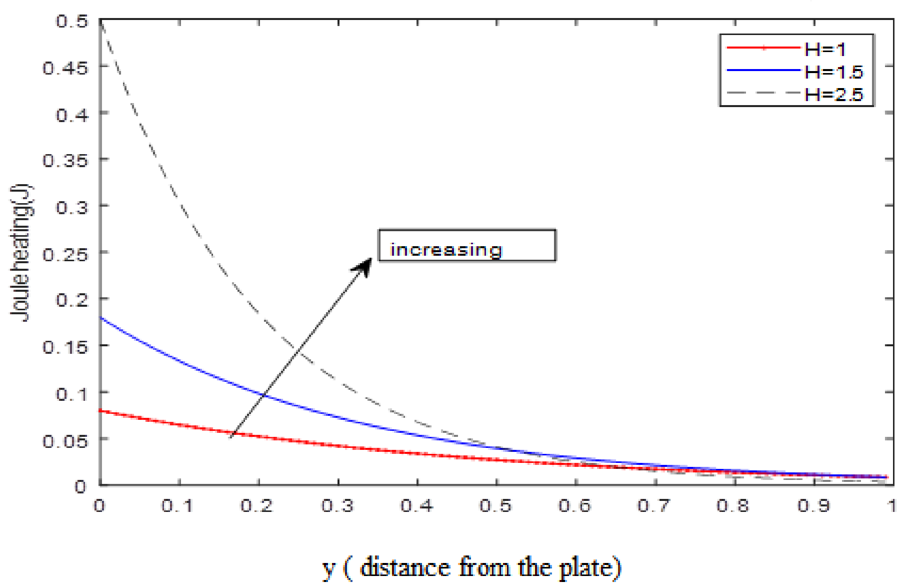


**Fig. 4 The effect of Hartmann Number on Joule dissipation**

## 7. CONCLUSION

In this study, the Laplace transform technique has been used to solve the momentum equation of a MHD flow. An analysis of the solution has been done in order to investigate the effect of Hartmann number on transient period of velocity, viscous dissipation and Joule heating. Graphical representation has also been done in order to study the effect of Harmann number on viscous and Joule heating. Through this study we have found that

- Transient period is shortened when Hartmann number is increased

- Viscous and Joule heating are increased when the Hartmann number is increased. This means that more heat is added in the fluid when Hartmann number is increased
- Velocity, viscous and Joule heating reach their steady state at the same time
- The steady state solution for Joule Heating and Viscous dissipation is the same.

This study reveals that an otherwise perpetually unsteady flow can be made steady by exposing the fluid to a weak magnetic field.

## REFERENCES


A. K. Thankal, R.N Jat and Deepak Kumar (2017). Unsteady MHD flow and heat transfer over porous stretching plate. International Journal of computational and Applied Mathematics.

C. Mamaloukas (2001). Numerical solution of the flow of a viscous conducting fluids produced by an oscillating plane wall subjected to a transverse magnetic field. Proceeding of the conference of Applied Differential Geometry-general relativity and the worship on Global Analysis, Differential geometry and lie Algebras, 87-94, © 2004 Balkan Society of Geometers Balkan press.

M. Abdulhameed, H Saleh, I Hashim and R. Roslan (2016). The transient MHD flow generated by a periodic wall motion in a porous space; Journal of Applied Fliud Mechanics, Vol. 9, No. pp 509 – 517.

M. A Hamad, I. Rop (2011). Unsteady MHD free convection flow past a vertical permeable flat plate in a rotating frame of reference with constant heat source in a nanofluid. Heat mass transfer 47: 1517-1524 DoI 10.10007/s00231 – 011 – 0816-6.

M.S Tillack and N.B Morley (1998). Magneto-hydrodynamics, McGraw Hill.

O. D Makinde and P.Y Mhome (2005). Heat Transfer to MHD oscillatory flow in a channel with porous medium. Rom Journal, Physics, Vol. 50, Nos, 9-10 P. 931-938, Bucharest.

P. M Kishore, V. Rajesh, S. Vijayakumar Verma (2012). The effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions. Theoret. Appl Mech., Vol. 39, no.2 pp. 99-125, Belgrade.

R. C Chaudhary and Arpita Jain (2007). Combined heat and Mass Transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium. Rom. Journal of Physics. Vol. 52, Nos 5-7 pp. 5015 – 524, Bucharest.

S. Ahmed and K. Kalita (2013). Analytical and Numerical study for MHD radiating flow over an infinite vertical surface Bounded by a porous medium in presence of Chemical Reaction. Journal of Applied Fluid Mechanics vol. 6, No.4 pp. 597 – 607.