A modeling Approach for the Analysis of Transmission Dynamics of Anthrax inAnimals

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Abstract

This paper seeks to develop a SIR model with vaccination compartment in the study of anthrax transmission dynamics in animal population. The model employ ordinary differential equations in the formulation of the model's equation. The model's steady states solutions are investigated. The disease free equilibrium and endemic equilibrium of the model are analyzed qualitatively. Vaccination rate below a certain critical value causes the anthrax disease to persist. Recruitment and contact rates are the most sensitive parameters that contribute significantly to the basic reproductive ratio.

1.0 Introduction

Anthrax is an infectious disease categorized under zoonotic diseases caused by a bacterium called *Bacillus anthracis* [4].Anthrax is a disease that affects both animal and human population. This disease is found naturally in soil [18] and mostly affects herbivores as compared to carnivores. Anthrax is one of the major diseases that cause uncontrolled mortality in cattle, pigs, sheep, goats and horses worldwide [1, 14, 19, 20]. Animals easily get infected with anthrax through contact with infected animals, consumption of infected grass or water and by inhalation of anthrax spores [2].The environment is usually infected with carcasses of infected animals. Grass and soil become the reservoirs of anthrax spores which can persist in the soil or grass for an extended period of time even under very extreme weather and environmental conditions. According to authors [13, 17, 21], anthrax can be prevented from being transmitted in animals through vaccination. This can be easily achieved by treating infected animals or vaccinating animals joining the herd or recovered animals. Treated animals do acquire temporal immunity which does not last long.Therefore there is need to vaccinate all live animals against anthrax disease to prevent transmission. The infective animals will show clinical symptoms which take time to manifest in the animals because of the incubation period of anthrax being about 3-8 days before they succumb to death caused by anthrax infection.

Authors in [22] model considered four compartments: Susceptible, Contamination, Infective and Pathogens. This model regards infective compartment level key to the transmission of anthrax in animals. In this model, the infected animals do show clinical signs of the disease that can be transmitted to susceptible animals. This model is an extension of work done by author [11].According to author [11], the model considers three compartments namely: Susceptible, Contamination and Pathogens. The model does not consider infective compartment to be key in the transmission of anthrax disease. According to the model, the infective compartment is considered to have very low reproductive ratio [4, 15] and does not cause any infections in animals.

Research done by authors [6], the model considers four compartments: Transmission, Carcass ingestion, Environment and Migration as possible means through which anthrax is transmitted in animals. In this model, carcass ingestion and removal of carcasses from the environment does not cause any decline of anthrax transmission in animals. This model is an extension of work done previously by authors [8] whose model only considered two compartments: Environment and Contamination. In the study done by authors [14], their model considered seven compartments. This model study the transmission dynamics of anthrax disease between animal and human populations. Their model considers sensitivity analysis and how each of the parameters contributes on the model.

2.0 The model

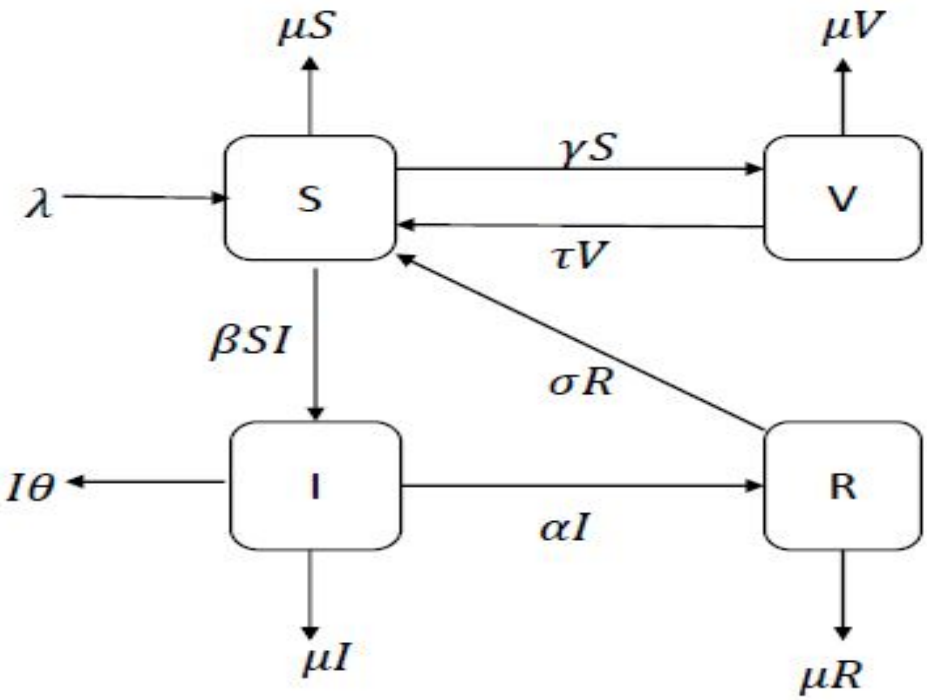
In our study model, we have employed part of the model constructed by authors [14] to investigate the effectiveness of constant vaccination policy on SIR model. Constant vaccination policy is an effective disease control mechanism that ensures that all animals joining the herd are vaccinated to prevent the transmission of the disease. This model also seeks to address the impact of recruitment and contact rates in the transmission dynamics of anthrax in animals. This was found not to have been captured by authors [14].This model also investigates the impact of disease-induced death rate in the animal population.However,this model cannot fully guarantee that all intruding and susceptible animals are vaccinated because of their large numbers and that the animal population interaction is not in a closed system.

This model was derived from the fact that most of our games reserves and national parks, human involvement in the transmission of anthrax is minimal though there are cases of intrusion by poachers killing animals for animal skin and tusks. Assuming human contribution to be negligible, it becomes significantly important to investigate the transmission of anthrax disease in animal population only.

This framework considers four compartments: Susceptible, Infective, Recovered and Vaccinated. The total animal population is divided at any time (t) into the four compartments with respect to their disease status in the system. The total animal population is given by N (t) =S(t)+I(t)+R(t)+V(t) where S (t) represents animals at risk of developing anthrax infection, I (t) represents animals showing anthrax symptoms, R (t) represents animals recovered from anthrax infection and acquired temporal immunity and V (t) represents animals vaccinated against anthrax attack.

The parameters used in this model are: λ denotes recruitment rate; β denotes contact rate; μ denotes natural death rate; γ denotes vaccination rate; τ denotes waning immunity of vaccinated animals; σ denotes waning recovery rate; θ denotes disease induced death rate and α animal recovery rate. These variables and parameters are all non-negative part.

The diagram below shows SIR model flow chart with vaccination compartment for anthrax transmission in animal population.



. Figure 1: SIR Flow chart with vaccination compartment

The model equations are non-linear ordinary differential equations given by:

$$\frac{dS}{dt} = \lambda - \beta SI - (\mu + \gamma)S + \sigma R + \tau V \quad (1)$$

$$\frac{dI}{dt} = \beta SI - (\mu + \theta + \alpha)I \quad (2)$$

$$\frac{dR}{dt} = \alpha I - (\mu + \sigma)R \quad (3)$$

$$\frac{dV}{dt} = \gamma S - (\mu + \tau)V \quad (4)$$

Equation (1) describes the dynamics of the susceptible animals. Equation (2) describes the nature of infected animals from anthrax disease. Equation (3) describes the dynamics of recovered animals after undergoing treatment and equation (4) describes the dynamics of vaccinated animals with anthrax vaccine.

3.0 Disease Free Equilibrium

Disease Free Equilibrium is given by $\varepsilon^0 = (S^0, I^0, R^0, V^0)$. At Disease Free Equilibrium there exists no anthrax disease hence no animals are infected with anthrax. According to authors in [5, 10], the dynamical systems of the equations (1)-(4) are equated to zero to determine the equilibrium point.

Taking initial conditions S (0)= S^0,I(0)=0,R(0)=0,V(0)=0 yields:

From equation (1), $S^0 = \frac{\lambda}{\mu + \gamma}$ (5)

Therefore, at disease free equilibrium, $\varepsilon^0 = (\frac{\lambda}{\mu + \gamma}, 0, 0, 0)$. (6)

4.0 Basic Reproductive Ratio (R_0)

According to authors in [4, 15], the basic reproductive ratio (R_0) can be found using Jacobian matrix J of ordinary differential equations (1)-(4) differentiated partially to yield:

$$\text{J (SIRV)} = \begin{pmatrix} -(\mu+\gamma) & -\beta\dfrac{\lambda}{\mu+\gamma} & \sigma & \tau \\ 0 & \beta\dfrac{\lambda}{\mu+\gamma}-(\mu+\theta+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & 0 & -(\mu+\tau) \end{pmatrix} \quad (7)$$

From the Jacobian matrix, one of the eigenvalues is $-(\mu+\gamma)$ .The rest of the eigenvalues can be found using matrix A given by:

$$\text{A} = \begin{pmatrix} \beta\dfrac{\lambda}{\mu+\gamma}-(\mu+\theta+\alpha) & 0 & 0 \\ \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & -(\mu+\tau) \end{pmatrix} \quad (8)$$

The determinant of A denoted as $\det(\text{A}) = \left(\beta\dfrac{\lambda}{\mu+\gamma}-(\mu+\theta+\alpha)\right)(\mu+\sigma)(\mu+\tau)$

$$= (\mu+\sigma)(\mu+\tau)\left[\beta\frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}-1\right] \quad (9)$$

From (9), the basic reproductive ratio $R_0 = \beta\dfrac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}$ (10)

Theorem 1

Disease free equilibrium point is locally asymptotically stable if $R_0 < 1$ and is unstable if $R_0 > 1$.

Proof

Disease free equilibrium point is given as $\varepsilon^0 = \left(\dfrac{\lambda}{\mu+\gamma}, 0, 0, 0\right)$ and basic reproductive ratio given as

$$R_0 = \beta\frac{\lambda}{\mu+\gamma}-(\mu+\theta+\alpha)$$

At disease free equilibrium $\beta\frac{\lambda}{\mu+\gamma}-(\mu+\theta+\alpha)\ <0$ (9)

Equation (9) can be expressed as $\beta\frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}-1<0.$

$$R_0=\beta\frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}$$

Therefore, $R_0-1<0$ which implies that $R_0<1.$

Given that $R_0<1$, disease free equilibrium point exists and is locally asymptotically stable.

Lemma 1

If $R_0>0$, then it follows that $\beta\frac{\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}-1>0$

Therefore, $R_0>1$ which implies that the disease free equilibrium is locally asymptotically unstable.

5.0 Endemic Equilibrium

The Endemic Equilibrium state is where the anthrax disease persist to be eradicated in animal population. The susceptible, infected, recovered and vaccinated compartments must not be zero at this equilibrium point. According to authors in [9], endemic equilibrium of dynamical systems (1)-(4) is given by $\varepsilon^*=(S^*,I,^*R,^*V^*)$ where $S^*>0, I^*>0, R^*>0$ and $V^*>0.$

From (7), the rest point becomes:

$$S^*=\frac{\mu+\theta+\alpha}{\beta}\ ,\quad I^*=\frac{(\mu+\gamma)(\mu+\tau)(\mu+\theta+\alpha)-\gamma\tau(\mu+\theta+\alpha)-\beta\lambda(\mu+\sigma)}{\beta(\mu+\tau)\left[\sigma\tau-(\mu+\sigma)(\mu+\theta+\alpha)\right]}(\mu+\sigma)$$

$$R^*=\tau\frac{(\mu+\gamma)(\mu+\tau)(\mu+\theta+\alpha)-\gamma\tau(\mu+\theta+\alpha)-\beta\lambda(\mu+\tau)}{\beta(\mu+\tau)\left[\sigma\tau-(\mu+\sigma)(\mu+\theta+\alpha)\right]},\ V^*=\gamma\frac{(\mu+\theta+\alpha)}{\beta(\mu+\tau)} \qquad (10)$$

Theorem 2

If $\gamma<\frac{\beta\lambda}{(\mu+\theta+\alpha)}-\mu$, the vaccination threshold is less than a certain critical value and endemic equilibrium point become unstable.

Proof

From (7) the Jacobian matrix J= $\begin{pmatrix} -(\mu+\gamma) & -\frac{\beta\lambda}{\mu+\gamma} & \sigma & \tau \\ 0 & \beta\frac{\lambda}{\mu+\gamma}-(\mu+\theta+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & 0 & -(\mu+\tau) \end{pmatrix}$

If the determinant is greater than zero

$$\frac{\beta\lambda}{(\mu+\gamma)}-(\mu+\theta+\alpha)\ >0 \qquad (11)$$

Re-arranging (11) yields

$$\frac{\beta\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}-1\ >0 \qquad (12)$$

But from (10), $R_0=\frac{\beta\lambda}{(\mu+\gamma)(\mu+\theta+\alpha)}$

Therefore, $R_0\ >1$. (13)

The endemic equilibrium will only occur if $R_0\ >1$. This means that the disease become unstable and the rest point is lost. The vaccinated animals loose their immunity and become susceptible.

Lemma 2

If $\gamma\ >\frac{\beta\lambda}{\mu+\theta+\alpha}-\mu$, the endemic equilibrium point becomes stable. Anthrax disease persists in animal population

6.0 Sensitivity Analysis and Baseline values

Sensitivity Analysis is used in determining how the parameters contribute to the basic reproductive ratio R_0 in the model. The table below shows sensitivity index and baseline values of each parameter and how it contributes on the model. Sensitivity analysis is given by the relation

$S_A^{R_0}=\frac{\partial R_0}{\partial A}\times\frac{A}{R_0}$ .Where A is any parameter used on the model.

Parameter	Contribution	Baseline values	References
λ	Positive	200	[12]
β	Positive	0.0001	[8,22]
μ	Negative	0.001	Estimate
γ	Negative	0.10	[8]
σ	0	0.02	Estimate
τ	0	0.003	[14]
θ	Negative	0.15	[22]
α	Negative	0.01	[8]

7.0 Results and Discussion

In this study, we modeled vaccination compartment in the transmission dynamics of anthrax in animal population. The outcome of stability analysis of the endemic equilibrium state shows that it is possible to effectively control anthrax transmission in animal population by vaccinating the animals.

In order to study this we take the initial conditions for endemic equilibrium $\varepsilon^*(\ S^* = 2000, I^* = 100, R^* = 300, V^* = 500)$ and period time (t) of 0-10 years. Parameters baseline values from other published literature have been used as indicated above. By using Matlab software (odesolve) the following results are obtained to investigate the impact of the parameters on R_0 and whether anthrax will be eradicated or will persist in the animal population.

Figure 3a

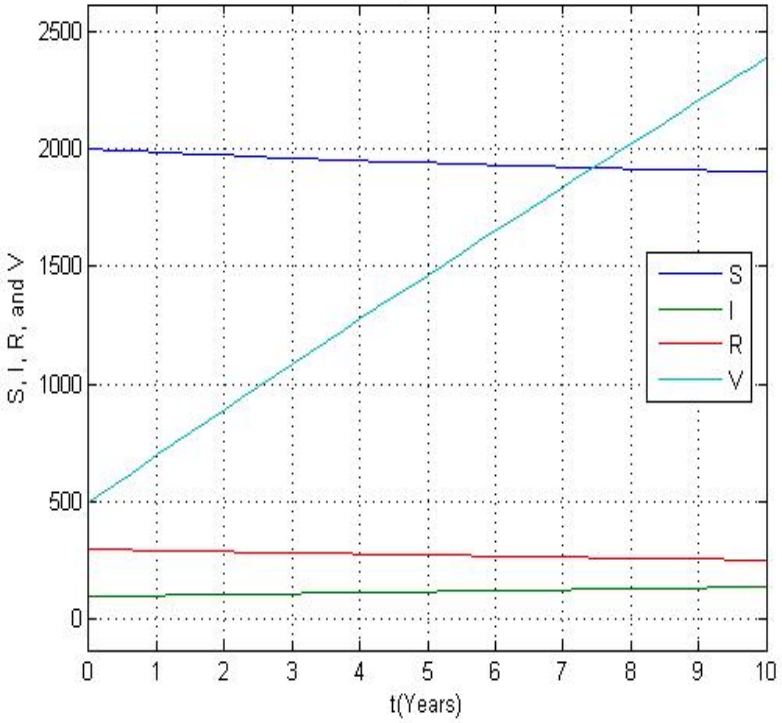
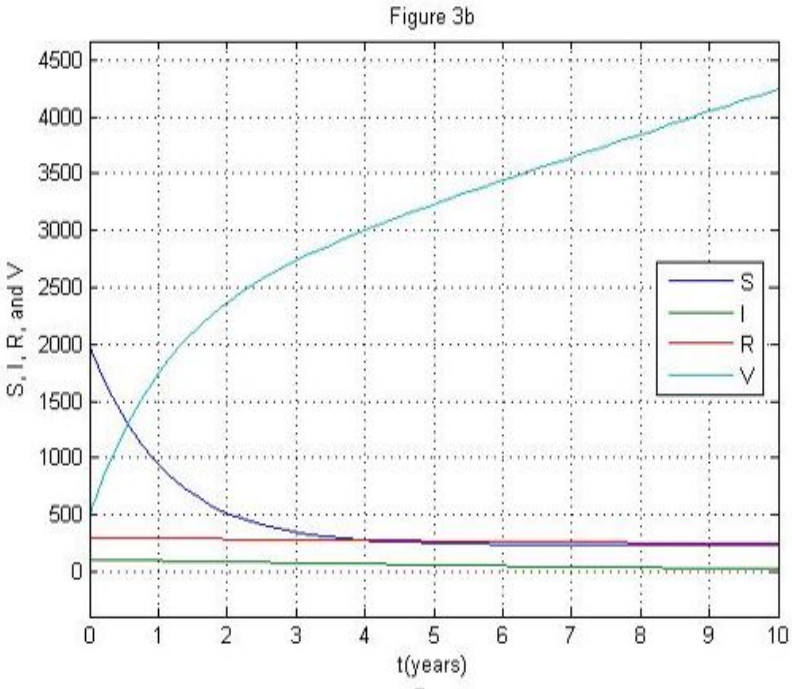
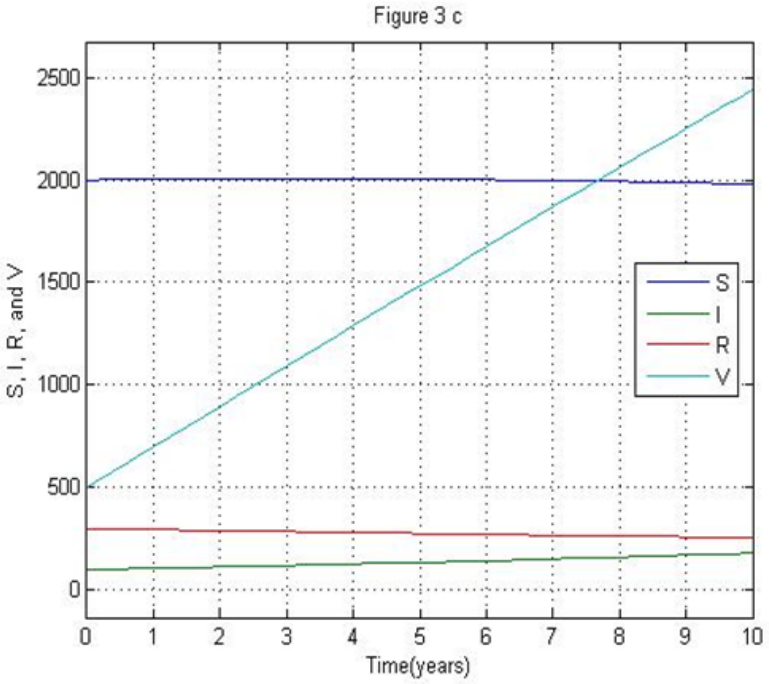


Figure 3b





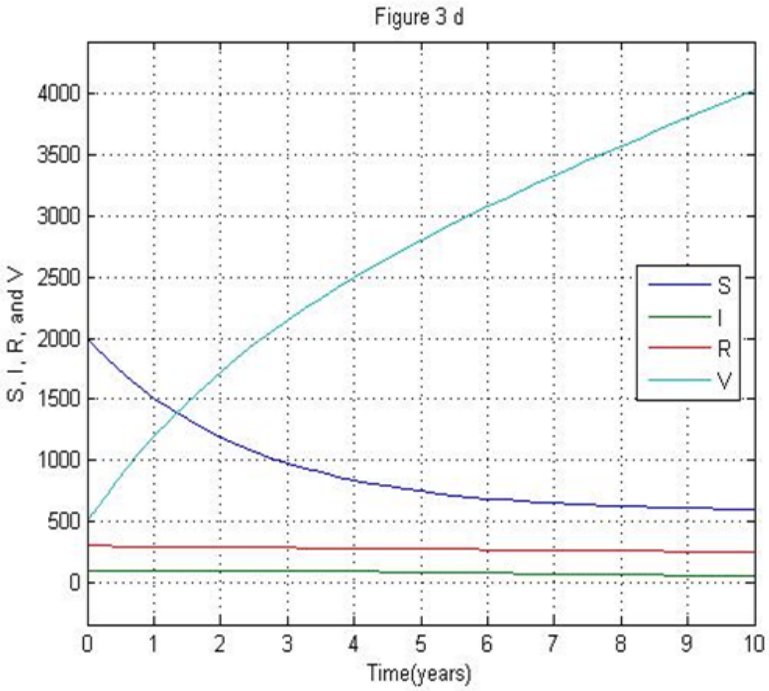


Figure 3 a: Graphical illustration for parameter values $\lambda = 200, \beta = 0.0001, \mu = 0.001$

$\gamma = 0.1, \sigma = 0.02, \tau = 0.003, \quad \theta = 0.15, \alpha = 0.01 \quad R_0 = 1.1648$

Figure 3 b: Graphical illustration for parameter values $\lambda = 200, \beta = 0.0001, \mu = 0.001, \gamma = 0.90$

$\sigma = 0.02, \tau = 0.003, \theta = 0.15, \alpha = 0.01 \quad R_0 = 0.1379$

Figure 3 c: Graphical illustration for parameter values $\lambda = 220, \beta = 0.00011, \mu = 0.001, \gamma = 0.10$

$\sigma = 0.02, \tau = 0.003, \theta = 0.15, \alpha = 0.01 \quad R_0 = 1.4882$

Figure 3 d: Graphical illustration for parameter values $\lambda = 220, \beta = 0.00011, \mu = 0.001, \gamma = 0.40$

$\sigma = 0.02, \tau = 0.003, \theta = 0.15, \alpha = 0.01 \quad R_0 = 0.3550$

The results from figures 3a and 3c shows that the susceptible animals are highly infected by the anthrax and transmission of the disease remain significantly high. This is because the vaccination rates are very low (that is $\gamma = 0.10$). These two graphs show evidence of endemic equilibrium state. Results from figures 3b and 3d show scenarios where the recruitment and contact rates have been increased from $\lambda = 200$ to 220 and $\beta = 0.0001$ to 0.00011 respectively. The vaccination rates are increased from $\gamma = 0.10$ to 0.4 and $\gamma = 0.10$ to 0.9 respectively. The number of susceptible animals significantly decreased due to high rate of vaccination. These results show evidence disease free equilibrium where the disease can be totally be eradicated from the animal population. The reproductive ratios R_O for figures 3a and 3c are above a unit (1.1648 and 1.4882) and for figures 3b and 3d the reproductive ratios are less than a unit (0.1379 and 0.3550)respectively. Increasing the rate of vaccination γ , the reproductive ratio R_O decreases. Therefore, animals will not die as a result of anthrax infection. When γ is decreased, the reproductive ratio R_O increases.

8.0 Recommendations

The outcome of the model shows that vaccination is a good control strategy against anthrax outbreak in animal population. However, vaccination may not completely guarantee protection of the animals against anthrax but it is possible that the vaccinated animals with time may lose immunity and may contract anthrax disease again. Therefore, there is need to keep vaccinating animals periodically against anthrax to prevent anthrax transmission in animals as low as possible.

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