Higher-order Spectral Filtering Effects on the Evolution of Stationary Dissipative Solitons

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Abstract A semianalytical method to study the effects of the higher-order spectral filtering in the cubic-quintic complex Swift-Hohenberg equation (CSHE) through the dynamics of one soliton was applied. The approach is based on a reduction from an infinite-dimensional dynamical dissipative system to a finite-dimensional model. This formulation is helpful to study the ground state of the soliton dynamic since it depends on a trial function and a good set of parameters. With real coefficients, the CSHE exhibits stationary dissipative solitons in space with the equation parameters, and the higher-order spectral filtering has a real impact on the cartographies of stationary soliton domain. The detailed analysis reveals the effects of spectral filtering term on the stationary soliton parameters, and displays that it differently influences the cubic and quintic terms of the CSHE. The results highlight the major influence of the spectral filtering on the temporal width of the stationary soliton whereas it does not have a real impact on the amplitude and the spatial width.

Keywords: *dissipative solitons, spectral filtering, fiber lasers, cubic-quintic Swift–Hohenberg equation, spectral response, gain spectrum*

1. Introduction

The laser systems with nonlinearity, saturable absorber and allowing the generations of ultra-short optical pulses, exhibit a variety of pulse shapes and evolutions. They perform a complicated dynamic, which distinguish them from the Hamiltonian soliton. This results from the fact that, in addition to dispersion and nonlinearity, the optical pulses include energy exchange with external sources. Thus, these laser cavities can be regarded as perfect surroundings for the concept of dissipative solitons and an ideal experimental frame for the exploration of dissipative soliton dynamics. In these systems with gain and loss, the soliton solutions appear as a result of a balance between dispersion (diffraction) and nonlinearity, then gain and loss must be also balanced.

In spite of the complexity of most spatially extended laser systems, many of them have been shown to be described by the cubic-quintic complex Ginzburg-Landau equation (CGLE) [1]. In the field of nonlinear optics, the CGLE can be used to describe a wide range of systems [2], such as passively mode-locked lasers with fast saturable absorbers, parametric oscillators, wide aperture lasers, nonlinear optical transmission lines [3] and nonlinear cavities with external pump [4]. It is undoubtedly that one can use just this equation to explain complicated phenomena in various systems. Hence, the CGLE has been intensely studied in many research studies [5, 6] and revealed a rich variety of solutions: stationary, pulsating, creeping, and erupting solitons [7].

In laser systems with a fast saturable absorber, a cubic-quintic complex Ginzburg-Landau equation is a suitable tool to study pattern formation. In these conditions, its quintic nonlinearity is essential to ensure the stability of optical pulses overcoming something that the cubic Ginzburg-Landau equation could not achieve. However, a cubic-quintic complex Ginzburg-Landau model is restricted to a second-order term and a spectral response with a single maximum, which is not the case in many experiments under real conditions. In these conditions, it is important to make the model more realistic and to take into account the situation when the gain spectrum is wide with multiple peaks. The addition of fourth-order spectral filtering term into the cubic-quintic Ginzburg-Landau equation leads to the complex Swift-Hohenberg equation (CSHE) and this is needed to depict optical pulses formation in wide aperture.

The CSHE plays the role of paradigms since it outlines the very basic mechanisms of pulse dynamic in many systems. It describes quite well a general theory of transverse patterns in wide aperture, single longitudinal mode lasers and synchronously pumped optical parametric oscillators. Under appropriate conditions, this equation depicts class A and C lasers [8, 9, 10]. Likewise, the CSHE representation of the two-level lasers has been extended to model semiconductor lasers [11] and has been inspected in [12, 13] to illustrate its validity in this context. A detailed account of the possible patterns present in the CSHE equation has been intensely studied via numerical studies [14, 15] and has been characterized using analytical techniques in the whole parameter space [16, 17].

Since, as the influence of spectral filtering on mode-locked fiber lasers is well known through several studies [18, 19], our purpose in the present study is to investigate the impact of the higher-order spectral filtering on the stationary solutions of the CSHE. In recent studies, the effect of spectral filtering in mode-locked fiber lasers with an extended geometrical model was presented [20]. It was demonstrated that the spectral filtering leads to strong nonlinear dynamics in a mode-locked fiber laser cavity, which can be used to understand the pulse dynamics in mode-locked soliton fiber lasers. Thus, the numerical models show the vital roles of the spectral filtering effects on chirped pulse behaviors [21].

In this paper, using chirped Gaussian pulse, the impact of the spectral filtering on the dynamic of the stationary soliton in the two-dimensional complex Swift-Hohenberg equation was analyzed. The remainder of the paper is organized as follows: first, in section 2 the governing equation and the collective variables approach are introduced and presented. The variational equations obtained from this semianalytical approach are then reported and analyzed. The section 3 is devoted to the investigation of the influence of the higher-order spectral filtering in the two-dimensional cubic-quintic complex Swift-Hohenberg equation through the dynamics of one soliton. Finally, in section 4, some concluding remarks are given.

2. Model and Analytical Study

In general, the cubic-quintic complex Swift-Hohenberg equation has complex coefficients and hence time-dependent solutions. Nevertheless, in the present paper, we restrict our attention to an important but special case of this equation, namely the case of real coefficients and two spatial dimensions. The analysis of this equation reveals a great variety of patterns and structures. It can be both quantitatively and qualitatively described as a number of nonlinear effects that occur during the optical pulses propagation. The CSHE characterizes also passively the mode-locked lasers that allow the generation of self-shaped ultra-short pulses in a laser system [22], and semiconductor laser [23]. The CSHE can be read in this normalized form [24, 25]:

$$\psi_z - iD\psi_{tt}/2 - i\psi_{rr}/2 - i\gamma|\psi|^2\psi - i\nu|\psi|^4\psi = \delta\psi + \varepsilon|\psi|^2\psi + \beta\psi_{tt} + \mu|\psi|^4\psi + \gamma_2\psi_{ttt} \qquad (1).$$

Where z (treated as a continuous variable) is the propagation distance or cavity round trip number, t is the retarded time in a frame of reference moving with the pulse and r $\left(r=\sqrt{x^2+y^2}\right)$ represents the transverse coordinate, taking account of the spatial diffraction effects. Here, $\psi=\psi(r,t,z)$ describes the complex amplitude of the transverse electric field, for example, inside a cavity.

The equation (1) without the additive term $\gamma_2\psi_{ttt}$ is the same as the cubic-quintic complex Ginzburg-Landau equation. In this study, the coefficients μ, δ, β, D, ν, γ, γ_2 and ε are real constants, the right-hand-side of equation (1) contains the dissipative terms and the left-hand side holds the conservative terms. $\delta, \varepsilon, \beta$ and μ are the coefficients for linear loss (if negative), nonlinear gain (if positive), spectral filtering (if positive) and saturation of the nonlinear gain (if negative), respectively. $D = +1(-1)$ is for the anomalous (normal) dispersion propagation regime and ν represents, if negative, the saturation coefficient of the Kerr nonlinearity. γ stands for Kerr nonlinearity coefficient. In our study, the dispersion is anomalous, and ν is kept relatively small. Finally, γ_2 which is of major significance for this present study represents the higher-order spectral filter term. The parameter γ_2 must be positive to have stable pulses in the frequency domain. The effect of the spectral filter can be described by the following transfer function:

$$T(\omega) = exp(\delta - \beta\omega^2 - \gamma_2\omega^4) \qquad (2).$$

When $\gamma_2 = 0$ (which corresponds to the CGLE, the spectral response is a Gaussian curve with amplitude δ and width β, and has a single maximum) [26]. But when γ_2 is nonzero, the response of the spectral filter is much more affected and gives a spectral response with two distinct maximums. In this case the spectral response depends on the values of γ_2 as one can see in Figure 1. Whatever the values of the higher-order spectral filter term $(\gamma_2)$are, the evolutions present two maximums but have a common local minimum. The amplitudes of these maximums depend on the γ_2 values. Maximum spectral responses evolve in a manner that is inversely proportional to the γ_2 parameter values. The less γ_2, the higher the maximum response.

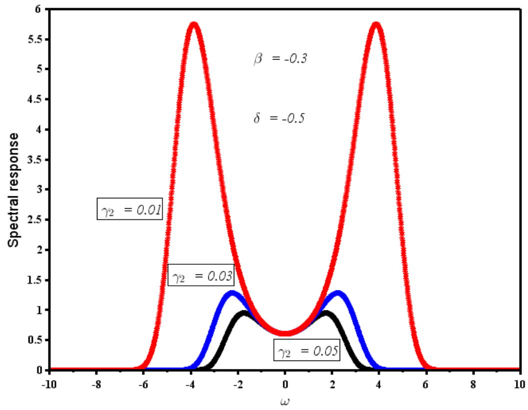

Figure 1. Spectral responses evolution according to the values of the higher-order spectral filter term γ_2. Red: $\gamma_2 = 0.01$, blue: $\gamma_2 = 0.03$, and black: $\gamma_2 = 0.05$

In our previous studies [16, 26, 27], using the collective variables theory [28] with some effectiveness, the stationary two-dimensional solutions of the CSHE were investigated. Indeed, as the 2D CHSE has several parameters that define the existence of stationary solutions, it is a tedious work to find the different types of the soliton according to the set of the equation parameters. To overcome this difficulty, one can use the master equation approach which helps to reduce an infinite-dimensional to an ordinary differential equation. In fact, the collective variables method [28] is based on a trial function theory. The idea consists to associate collective variables with the pulse's parameters of interest for which equations of motion may be derived. The resulting dynamical system controls the evolution of a finite number of parameters such as the pulse amplitude, width, and chirp. This is the way to obtain a significant reduction in the number of variables used for the description of the soliton dynamics. To this end, one can decompose the optical field in the following way:

$$\psi(r,t,z) = f(x_1, x_2, \dots x_n, t) + q(z,t) \tag{3}$$

with f the trial function, dependent on the collective variables (x_n), and q the residual field that describes all other excitations in the system (radiation, dressing field, noise, etc.)

Using the bare approximation [28] to the 2D CSHE (for more details, see [16, 26, 27, 28]) and applying the following Gaussian function as ansatz function:

$$f = Aexp\left(-\frac{t^2}{w_t^2} - \frac{r^2}{w_r^2} + \frac{i}{2}c_t t^2 + \frac{i}{2}c_r r^2 + ip\right) \tag{4}$$

six collective variables that evolve according to the following set of six coupled ordinary differential equations are obtained.

$$\dot{A} = A\delta + \frac{3}{4}A^3\varepsilon - \frac{2}{w_t^2}A\beta + \frac{5}{9}A^5\mu - Ac_tD - 2Ac_r + 3\left(2c_t^2 - w_t^2c_t^4 + \frac{3}{w_t^4}\right)A\gamma_2,$$

$$\dot{w}_t = 2w_tc_tD - \frac{1}{4}w_tA^2\varepsilon - \frac{2}{9}A^4w_t\mu + (1 - w_t^4c_t^2)\frac{2\beta}{w_t} + (w_t^8c_t^4 - 1)\frac{12}{w_t^3}\gamma_2,$$

$$\dot{w}_r = 4w_rc_r - \frac{1}{4}w_rA^2\varepsilon - \frac{2}{9}A^4w_r\mu,$$

$$\dot{c}_t = 2\left(\frac{1}{w_t^4} - c_t^2\right)D - \frac{8}{w_t^2}c_t\beta - \frac{1}{2w_t^2}A^2\gamma - \frac{4}{9w_t^2}A^4\nu + 48c_t\left(\frac{1}{w_t^4} + c_t^2\right)\gamma_2,$$

$$\dot{c}_r = -4c_r^2 - \frac{1}{2w_r^2}A^2\gamma + \frac{4}{w_r^4} - \frac{4}{9w_r^2}A^4\nu,$$

$$\dot{p} = 2\beta c_t + \frac{3}{4}A^2\gamma - \frac{D}{w_t^2} - \frac{2}{w_r^2} + \frac{5}{9}A^4\nu - 12c_t\left(\frac{1}{w_t^2} - w_t^2c_t^2\right)\gamma_2 \tag{5}.$$

A, w_t, w_r, c_t, c_r and p are the collective variables and represent respectively the amplitude, the temporal and spatial widths of the soliton, the chirp along t axis, the spatial chirp and p the global phase. The collective variables (x_n) are variables that evolve along the propagation direction z and the dynamic of the dissipative soliton.

It is clearly see that the CSHE, equation (1) is reduced to an ordinary differential equation given by the soliton parameters A, w_t, w_r, c_t, c_r and p. As well, the collective variables method helps to show explicitly how each coefficient of the CSHE (equation 1) governs the soliton parameters (amplitude, widths, chirps and the global phase). A detailed analysis of the six coupled ordinary differential equations reveals that the spectral filter terms (γ_2 and β) do not affect the spatial width and the spatial chirp. However, the evolution of the temporal parameters (width and chirp), the amplitude and the global phase are influenced by the effects of the spectral filter. All in all, the impact of the higher-order spectral filter term is clearly highlighted with the help of ordinary differential equations. Through the equation (5) illustrate the role of the dissipative and conservative terms of the CSHE.

The collective variables approach's best asset lies in the fact that it reveals in detail the influence of each parameter of CSHE under soliton parameters. In addition, it makes it possible to express the total energy with respect to the soliton parameters, and gives a first idea on the dynamic of the pulse. Here, the total energy is given by the following equation:

$$Q = \frac{\pi\sqrt{2\pi}}{4}w_tA^2w_r^2 \tag{6}$$

showing that the energy relies solely on the amplitude of the soliton, its temporal and spatial widths.

3. Stationary Soliton under Influence of Spectral Filtering

The CSHE admits stationary solitons and we provide evidence for its localized solutions in the space parameters in the recent studies [26]. The stationary solutions correspond to the stable fixed points of the system, obtained from the ordinary differential equations. it proved that the collective variables approach is suitable for the procedure of derivation of the variational equations. It also provides the basic parameters of the fixed points, and helps to define the cartography of the stationary and the pulsating solutions [26, 27]. Indeed, as it is extremely difficult, if not impossible to vary all the parameters at the same time to find stable solutions, the ingenuity is to vary two parameters by setting all the others. Thus, one can easily obtain in a map all the different solutions.

With the following initial condition,

$$\psi(r,t,0) = 2.86exp\left(-\frac{t^2}{0.7} - \frac{r^2}{1.36}\right) \tag{7}$$

and investigating the parameter regions located in the vicinity of the parameters $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$ and $\gamma_2 = 0.05$, the stable stationary solutions in the (ν, ε) plane were illustrated. In fact, the behavior of the laser system is then largely determined by the gain ε and principally the cavity detuning ν, which is directly related to the difference between the atomic frequency and the closest cavity resonance frequency.

For a given set of ν and ε values, the use of the Newton-Raphson helps to find the corresponding fixed point before determining its stability. The mapping below (Fig. 2) shows the result of this rigorous analysis for the range of the selected values. The cartographies in red display the domains of stationary solitons for the set of parameters γ_2 while the other parameters remain constant. Here, one could specify that for dissipative systems, the total energy is not conserved but evolves in accordance with the so-called balance equation. So, when the stationary soliton is reached, the total energy converges to a constant value. Thus, each soliton parameter (amplitude and widths) in the stationary domain (in red) remains constant regardless of propagation distance.

A fine analysis of the Figure 2 a), b), and c), reveals that the higher-order spectral filter term values play an important role on the size of the stable stationary domain. It clearly appears that for the set of fixed parameters when the higher-order spectral filter term γ_2 decreases (from 0.05 to 0.03) the domain of stationary soliton (in red) widens. In this context, stationary solutions have a tendency to vanish gradually when γ_2 increases; which shows the importance of this parameter. The spectral filtering is therefore decisive in the formation of CSHE stationary solitons.

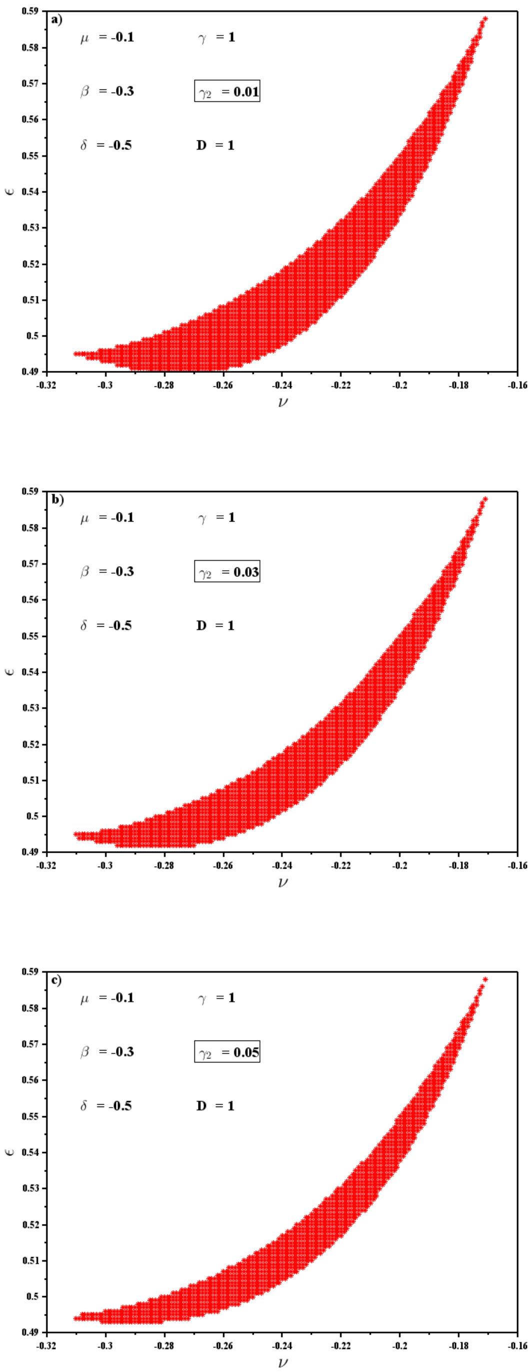

Figure 2. Cartographies of the solutions of the 2D complex Swift-Hohenberg equation in the (ν, ε) plane. The stable fixed points regions in red represent the domain of stationary solitons. Other CSHE parameters appear inside the **Figure 2.**

An investigation on the total energy evolution according to the nonlinear gain coefficient for different values of the higher-order spectral filter term γ_2 was performed. The Figure 3 shows this discussion clearly. It appears that the total energy increases in size as the nonlinear gain increases for a given γ_2 value.

This clearly reflects that for these stationary solitons, an increase in nonlinear gain lead to energy solutions. Furthermore, the higher-order spectral filter term γ_2 also plays an important role in the evolution of energy. In fact, the Figure 3 points out the evolution of the energies for the three values of γ_2 ($\gamma_2 = 0.01, \gamma_2 = 0.03$, and $\gamma_2 = 0.05$). One can clearly deduce that for small values of the nonlinear gain, the three curves are almost identical. However, when the nonlinear gain is greater than $\varepsilon \approx 0.504$, the energy evolutions are considerably distinguishable. The curves look the same. The energy increases with γ_2 value. It can thus notice that for these chosen values, the stationary solutions will have practically the same energy regardless of the values of γ_2, for the low values of the gain. But, when the nonlinear gain exceeds a threshold, the energy increases with the higher-order spectral filter term.

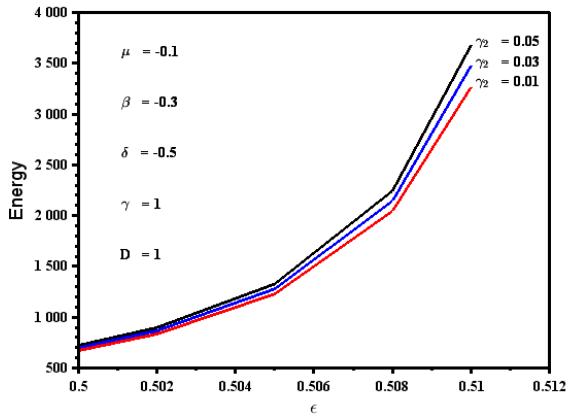

Figure 3. Evolution of the total energies of stationary dissipative solitons for different values of the higher-order spectral filter term. The red curve corresponds to $\gamma_2 = 0.01$, blue to $\gamma_2 = 0.03$, and black to $\gamma_2 = 0.05$. The values of other parameters are $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$ and $\nu = -0.252$.

As the nonlinear gain ε is a cubic term, we have tried to see how γ_2 acts on the quintic terms. Therefore, the evolution of the total energy according to the saturation coefficient of the Kerr nonlinearity for the same different values of the higher-order spectral filter term γ_2 was plotted. It notices that the appearance of the curves (Figure 3 vs Figure 4) is reversed. The Figure 4 shows distinctly that the total energy decreases in size as the saturation of the Kerr nonlinearity increases for a given value of γ_2. For low saturation coefficient of the Kerr nonlinearity values, the energy evolutions are well separated. More ν increases, the curves get closer without getting confused. For a given γ_2, the stationary soliton energy decreases when the quintic terms increases. The three curves keep the same appearance.

The energy increases with different increasing values of γ_2. It appears precisely that the higher-order spectral filter term acts differently on the actions of the cubic and quintic terms.

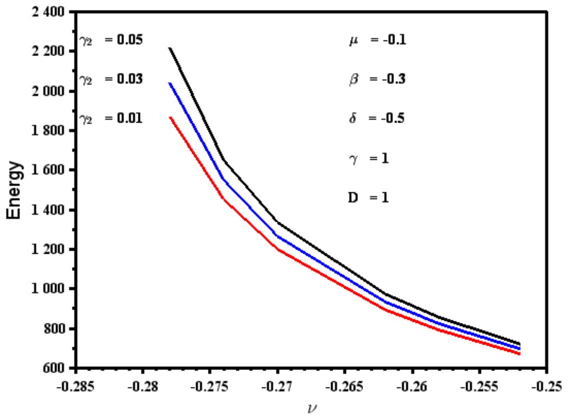

Figure 4. Evolution of the total energies of stationary dissipative solitons for different values of the higher-order spectral filter term. The red curve corresponds to $\gamma_2 = 0.01$, blue to $\gamma_2 = 0.03$, and black to $\gamma_2 = 0.05$. The values of other parameters are $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$ and $\varepsilon = 0.50$.

It emerges from the examination of Figures 3 and 4 that the higher-order spectral filter parameter γ_2 has a real impact on the dynamics of stationary solitons through the energies evolution. As it has seen above, the parameter γ_2 influences differently the cubic and quintic terms of the CSHE. To further investigate this study, it seemed appropriate to see how this same term (γ_2) affects the stationary soliton amplitude and widths. Indeed, one of the benefits of the collective variables approach is being able to follow (individually) the dynamic of soliton parameters namely its amplitude and widths. To highlight the action of the γ_2 factor on the propagation of the stationary soliton, on the Figures 5, 6 and 7, the evolution of the soliton amplitude and widths for the three values of γ_2 ($\gamma_2 = 0.01, \gamma_2 = 0.03$, and $\gamma_2 = 0.05$) while keeping the other terms constant and for fixed values of nonlinear gain ($\varepsilon = 0.508$) and saturation of the Kerr nonlinearity ($\nu = -0.252$) were plotted. A thorough interpret of these figures reveals that when γ_2 goes from 0.05 to 0.01, the amplitude of the stationary soliton is largely unchanged (≈ 2.1). Moreover, the spatial width is not influenced by this variation (≈ 5.9). This is quite in accordance with the ordinary differential equations (4). In fact, in these equations (5), the spatial width evolution ($\dot{w}_r$) doesn't contain any coefficient γ_2, so it has no influence on its propagation, which is in accordance with the Figures 5, 6 and 7. On the other hand, it is quite true that the amplitude equation ($\dot{A}$) contains the term γ_2, but its variation (from 0.05 to 0.01) has no significant effect on the amplitude propagation (Figures 5, 6 and 7). The γ_2 term being not preponderant with respect to other terms, its action turns out to be practically negligible. In addition, the study of the figures shows that the higher-order spectral filter γ_2 variation (from 0.05 to 0.01) dominates the dynamic of the temporal width. For $\gamma_2 = 0.05$, $\gamma_2 = 0.03$, and $\gamma_2 = 0.01$, the temporal width stays at $w_t \approx 7.1$, $w_t \approx 6.9$, $w_t \approx 6.7$, respectively. Therefore, the temporal widths remain constant but the amplitude dynamic changes for a given value of γ_2. This demonstration points out that the higher-order spectral filter γ_2 really affects the temporal width solely. Otherwise the contour plot confirms the studies and shows that the variations mainly concern the temporal widths. To conclude this section and confirm the results of the higher-

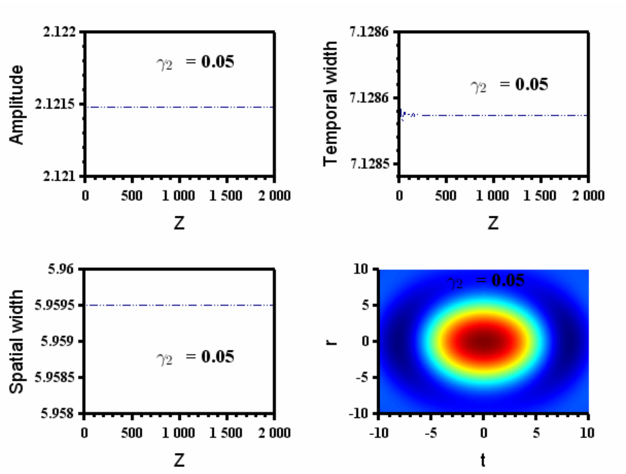

Figure 5. Evolution of stationary soliton amplitude, spatial and temporal widths for the higher-order spectral filter $\gamma_2 = 0.05$. The values of other parameters are $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$, $\nu = -0.252$ and $\varepsilon = 0.508$.

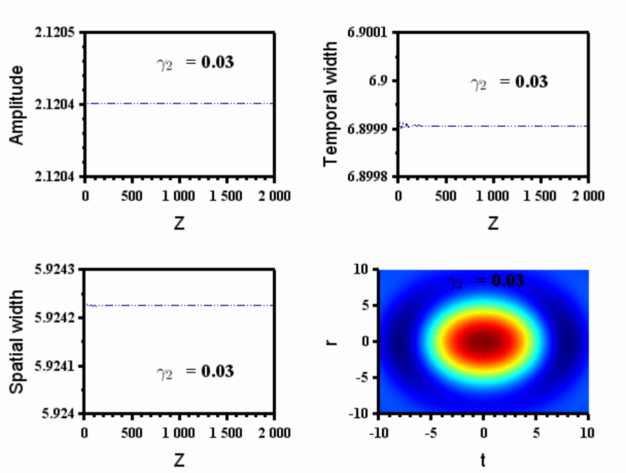

Figure 6. Evolution of stationary soliton amplitude, spatial and temporal widths for the higher-order spectral filter $\gamma_2 = 0.03$. The values of other parameters are $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$, $\nu = -0.252$ and $\varepsilon = 0.508$.

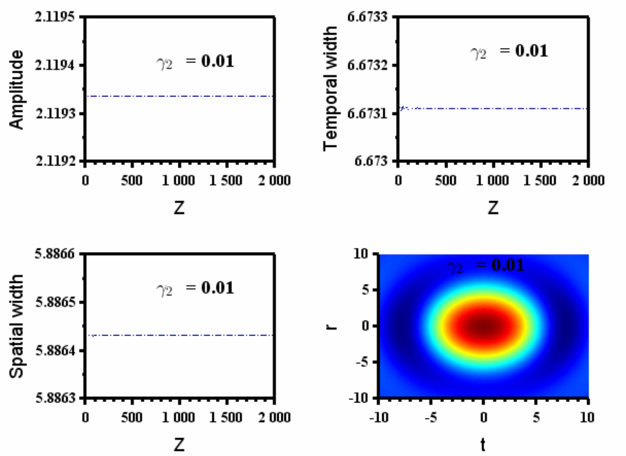

Figure 7. Evolution of stationary soliton amplitude, spatial and temporal widths for the higher-order spectral filter $\gamma_2 = 0.01$. The values of other parameters are $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$, $\nu = -0.252$ and $\varepsilon = 0.508$.

-order spectral filter γ_2 effect on the stationary solitons, the temporal and radial profiles of a stationary soliton for the three γ_2 values and for the values of nonlinear gain ($\varepsilon = 0.508$) and saturation of the Kerr nonlinearity ($\nu = -0.252$) were plotted. The fruit of these efforts is illustrated on the figures 8 and 9. It clearly appears that the higher-order spectral filter γ_2 has no effect on the amplitude and the spatial width of the stationary soliton (Figure 8). However, the temporal width changes for each value of the higher-order spectral filter γ_2 (Figure 9) but keeps the same amplitude.

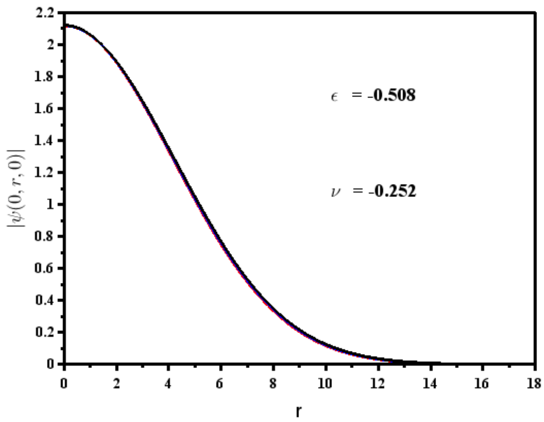

Figure 8. The radial profiles of the total stationary dissipative solitons for different values of higher-order spectral filter term. The red curve corresponds to $\gamma_2 = 0.01$, blue to $\gamma_2 = 0.03$, and black to $\gamma_2 = 0.05$. The values of other parameters are $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$, $\nu = -0.252$ and $\varepsilon = 0.508$.

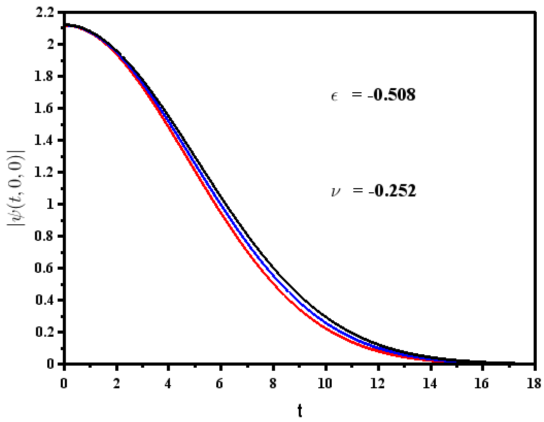

Figure 9. The temporal profiles of the total stationary dissipative solitons for different values of higher-order spectral filter term. The red curve corresponds to $\gamma_2 = 0.01$, blue to $\gamma_2 = 0.03$, and black to $\gamma_2 = 0.05$. The values of other parameters are $D = \gamma = 1$, $\beta = -0.3$, $\delta = -0.5$, $\mu = -0.1$, $\nu = -0.252$ and $\varepsilon = 0.508$.

4. Conclusion

In this paper, the effects of higher-order spectral filter term on the stationary dissipative soliton were investigated. The dynamical behavior of stationary soliton in the two-dimensional Complex Swift-Hohenberg equation under the spectral filtering was carried out. The domains of coexistence of stationary soliton are obtained through the semianalytical method, i.e., the collective variable approach. It appears in this study that the spectral filtering plays an important role in the formation of the stable stationary soliton. Therefore, the dissipative stationary solutions tend to vanish gradually when γ_2 increases. The detailed analysis points out that the spectral filtering also has a significant impact on the temporal width of the stationary profile while it does not really affect the amplitude and the spatial width. In addition, the parameter γ_2 influences differently the cubic and quintic terms of the 2D CSHE. Thus, when designing lasers, attention should be paid to the cubic and quintic parameters because they act differently on the spectral filtering.

To conclude, in this paper using a semianalytical approach with a suitable trial function, the influence of the spectral filtering on stationary soliton parameters have been demonstrated. It is hoped that these results can be extended to describe the pulsed operation in laser cavity and can be utilized to understand and engineer the pulse dynamics in mode-locked soliton fiber lasers.

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